

Math 21C Practice Midterm II Spring 2024

You may not use a calculator.

You may use one page of notes.

You may not use the textbook.

Please do not simplify answers.

1. (12 pts: Power Series)

Determine the x values for which the following power series converges:

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} (x-1)^n.$$

2. (11 pts for each part: Taylor Polynomials)

Find the first three nonzero terms for the following Taylor series associated to

(a) $f(x) = \sqrt{x}$ about $x = 4$

(b) $f(x) = \cos(\sqrt{x}) \sin(2x)$ about $x = 0$.

Hint: For this one you can use shortcuts and not compute any derivatives.

3. (11 pts: Taylor Remainder)

Estimate the error if the Maclauren polynomial $P_1(x) = 1 - x$ associated to $f(x) = \int_{t=0}^x e^{-\sin(t)} dt$ is used to make the estimate of $\frac{2}{3}$ for the integral $\int_{t=0}^{\frac{1}{3}} e^{-\sin(t)} dt$.

4. (11 pts: Vectors)

Let $\mathbf{u} = \langle 1, -2 \rangle$ and $\mathbf{v} = \langle 3, 4 \rangle$.

Find $\text{proj}_{\mathbf{v}} \mathbf{u}$.

5. (11 pts: Forces)

Consider a 100N weight suspended by two wires with slopes -1 and 2 .

Find the magnitudes of the force vectors on the two wires.

6. (11 pts: Lines)

Consider the following two intersecting lines: L_1 is given by $x = 1 + t$, $y = 2t$ and $z = -1 + 3t$. L_2 is given by $x = 3 + 2s$, $y = 1 + s$ and $z = -2 - s$.

(a) Find their point (x, y, z) of intersection.

(b) Find the angle between the lines.

7. (11 pts: Planes)

Compute the distance from the origin $(0, 0, 0)$ to the plane $2x + y - 2z = 6$.

8. (11 pts: Functions)

Consider the function $f(x, y) = 4 - \sqrt{y - x^2}$.

(a) Determine and sketch the domain of f in the plane.

(b) Determine the range of f .

9. (10 pts: Extra Credit... you may skip this problem)