

Math 21C

Kouba

Plane Vectors, Space Vectors

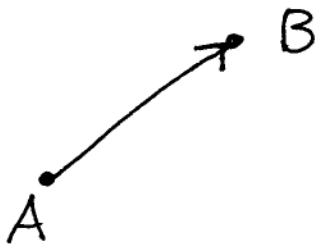
Def: A vector is a symbol representing direction and magnitude (length)

Ex: Vectors can describe

- 1.) speed and direction of fluid flow
- 2.) speed and direction of object moving along a path
- 3.) magnitude and direction of an applied force

Note: Two vectors are equal if they have the same direction and magnitude.

Notation: Vector \vec{AB} is a vector with initial point A and terminal point B.



Ex: Find vector \vec{AB} .

1.) $A: (1, 1)$, $B: (4, 3)$

2.) $A: (3, -1)$, $B: (-2, -4)$

$$3.) A: (0, 2), B: (3, 4)$$

$$4.) A: (1, 0, -1), B: (2, -3, 4)$$

$$5.) A: (3, -2, 2), B: (0, 0, 0)$$

Def: Let (\vec{a}, \vec{b}) and $(\vec{a}, \vec{b}, \vec{c})$ be vectors.
The magnitude of each vector is

$$1.) |(\vec{a}, \vec{b})| = \sqrt{a^2 + b^2}$$

$$2.) |(\vec{a}, \vec{b}, \vec{c})| = \sqrt{a^2 + b^2 + c^2}$$

Algebra of Vectors:

Let k be a constant.

$$1.) a.) (\vec{a}, \vec{b}) + (\vec{c}, \vec{d}) = (\vec{a+c}, \vec{b+d})$$

$$b.) k(\vec{a}, \vec{b}) = (k\vec{a}, k\vec{b})$$

$$2.) a.) (\vec{a}, \vec{b}, \vec{c}) + (\vec{d}, \vec{e}, \vec{f}) = (\vec{a+d}, \vec{b+e}, \vec{c+f})$$

$$b.) k(\vec{a}, \vec{b}, \vec{c}) = (k\vec{a}, k\vec{b}, k\vec{c})$$

$$3.) a.) \text{ If } (\vec{a}, \vec{b}) = (\vec{c}, \vec{d}), \text{ then } a=c \text{ and } b=d.$$

$$b.) \text{ If } (\vec{a}, \vec{b}, \vec{c}) = (\vec{d}, \vec{e}, \vec{f}), \text{ then } a=d, b=e, \text{ and } c=f.$$

$$4.) a.) |k(\vec{a}, \vec{b})| = |k| |(\vec{a}, \vec{b})|$$

$$b.) |k(\vec{a}, \vec{b}, \vec{c})| = |k| |(\vec{a}, \vec{b}, \vec{c})|$$

Def: any vector \vec{u} of length 1 is called a unit vector.

Ex: Show that each vector is a unit vector.

1.) $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ 2.) $(-1, 0)$

3.) $\left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ 4.) $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

Fact: Let \vec{w} be any vector.
A unit vector in the same direction as \vec{w} is

$$\vec{u} = \frac{1}{|\vec{w}|} \vec{w}$$

Ex: Make each vector a unit vector in the same direction.

1.) $(1, -1)$ 2.) $(3, 4)$

3.) $(0, 0, 3)$ 4.) $(-2, 3, -6)$

Geometry of Vectors

I.) Scalar Multiplication:

Let \vec{w} be a vector, k a constant