

Math 21C
Kouba
Surfaces of Revolution

PROBLEM : Consider the two-dimensional graph G of any equation in two variables, i.e., consider G to be a graph in the xy -plane, the yz -plane, or the xz -plane. Create a surface of revolution in three-dimensional space by revolving G around an axis (line) L . (Line L may be vertical, horizontal, or tilted.) We want to determine an equation for this surface using an arbitrary point (x, y, z) on the surface.

SOLUTION :

Step 1. Select a random point $P = (x, y, z)$ on the three-dimensional surface. The goal is to use these variables to write an equation which represents this surface.

Step 2. Determine the point Q , which

- a.) depends on point P ,
- b.) lies on the axis of revolution L ,

and

- c.) is nearest point P .

Step 3. Determine the point R , which

- a.) depends on point Q ,
- b.) lies on the original graph G ,

and so that

c.) points P , Q , and R are now part of a *cross-sectional circle* with Q at the center and segments PQ and QR forming radii of the circle.

Step 4. Use the distance formula to compute the lengths of PQ and QR . The equation can now be determined by setting

$$\text{length } PQ = \text{length } QR \quad .$$

