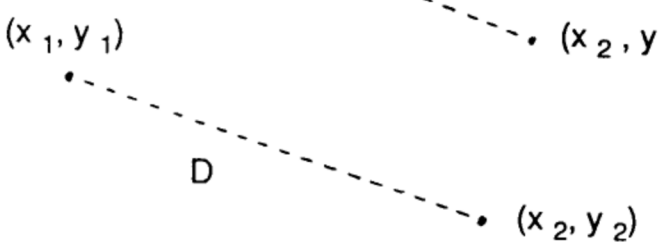
Math 21 C
Kouba
Three-Dimensional (3-D) Space

RECALL : Consider two points (x_1, y_1) and (x_2, y_2) in two-dimensional space. The <u>midpoint</u> of the line segment joining these two points is given by

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) .$$

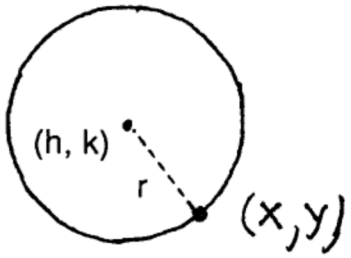
The <u>distance</u> between these two points is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} .$$



RECALL : The set of all points (x, y) in two-dimensional space which are a distance r from a fixed point (h, k) is a <u>circle</u> (with center (h, k) and radius r) given by the equation

$$(x-h)^2 + (y-k)^2 = r^2 .$$

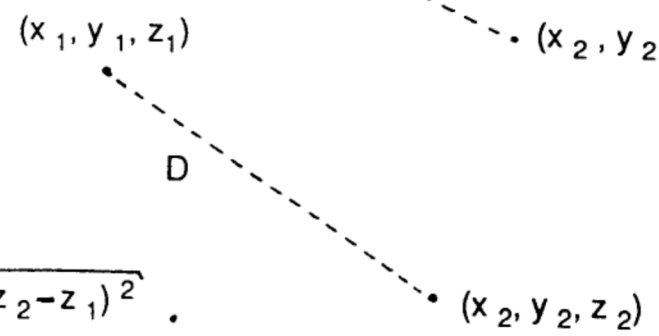


Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be two points in three-dimensional space. The <u>midpoint</u> of the line segment joining these two points is given by

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} , \frac{z_1 + z_2}{2} \right) .$$

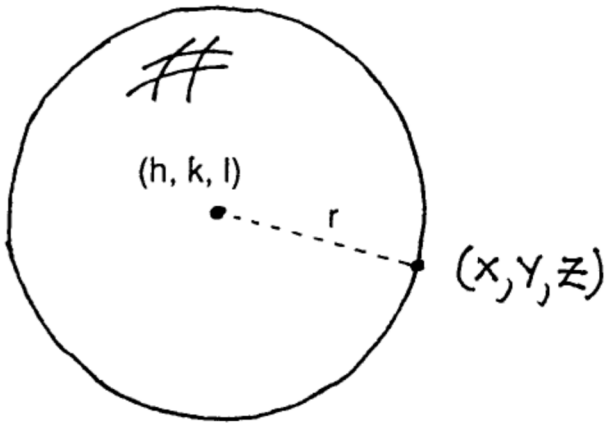
The <u>distance</u> between these two points is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} .$$



DEFINITION : The set of all points (x, y, z) in three-dimensional space which are a distance r from a fixed point (h, k, l) is a sphere (with center (h, k, l) and radius r given by the equation

$$(x-h)^2+(y-k)^2+(z-l)^2 = r^2.$$



Example : Find the center and radius of each of the following spheres.

1. $2x^2+2y^2+2z^2 = 32$

   center (0, 0, 0)
   radius 4

2. $x^2+y^2+z^2-4x+6y = 17$

   center (2, −3, 0)
   radius $\sqrt{30}$

Example : The diameter of a sphere has endpoints (1, 3, 0) and (-2, 4, 6). Determine an equation for this sphere.

$$(x+1/2)^2+(y-7/2)^2+(z-3)^2 = 23/2$$

Example : Find and simplify an equation for all points (x, y, z) in three-dimensional space which are equidistant from the point (1, −2, 3) and the plane $z = -1$.

$$z = 1/8\,(x-1)^2+1/8\,(y+2)^2+1$$