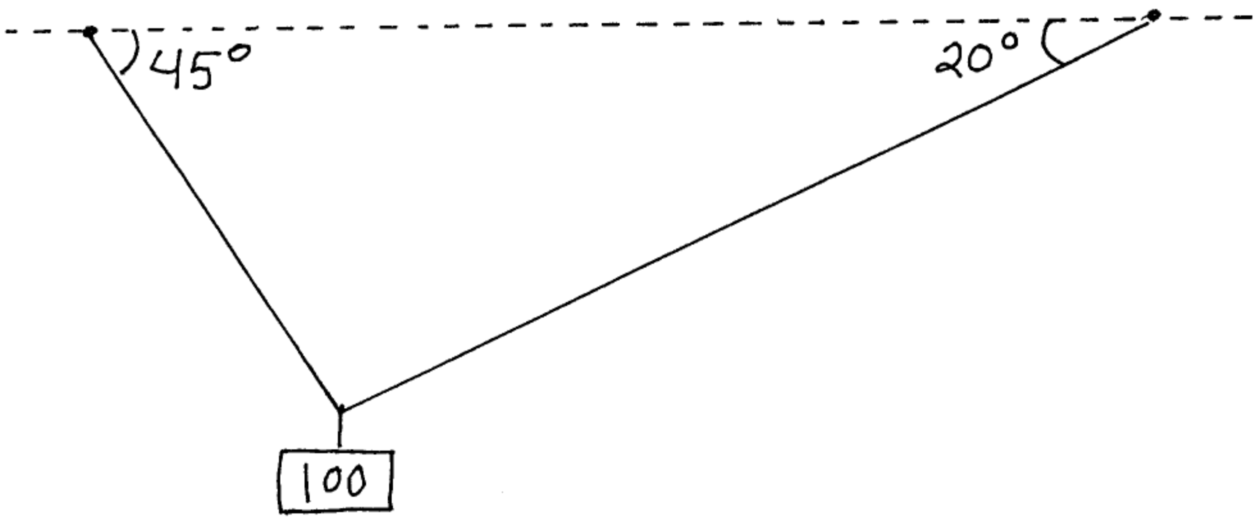
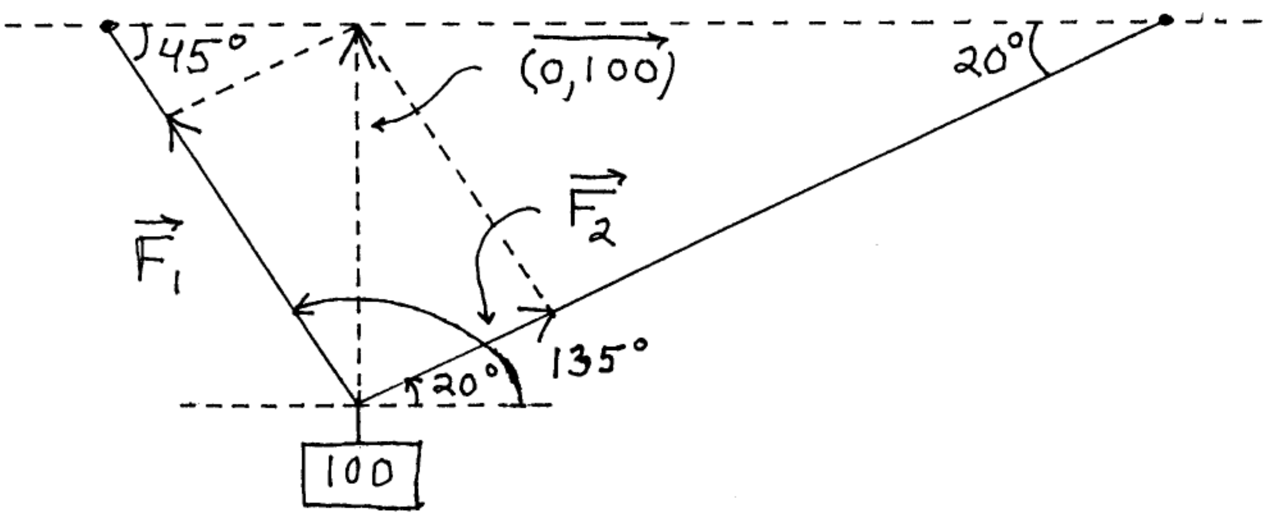
Math 21C
Kouba
An Example Using Force Vectors

Consider an object weighing 100 newtons which is suspended by two thin wires in the diagram below. Determine each force vector and their magnitudes.





Write vectors $\vec{F_1}$ and $\vec{F_2}$ in polar form :

$$\vec{F_1} = \overrightarrow{(|\vec{F_1}|\cos 135^\circ, |\vec{F_1}|\sin 135^\circ)}$$
$$= \overrightarrow{\left(|\vec{F_1}|\left(-\tfrac{\sqrt{2}}{2}\right), |\vec{F_1}|\left(\tfrac{\sqrt{2}}{2}\right)\right)},$$

$$\vec{F_2} = \overrightarrow{(|\vec{F_2}|\cos 20^\circ, |\vec{F_2}|\sin 20^\circ)};$$

since $\vec{F_1} + \vec{F_2} = \overrightarrow{(0,100)}$ we get that

$$\overrightarrow{\left(|\vec{F_1}|\left(-\tfrac{\sqrt{2}}{2}\right) + |\vec{F_2}|\cos 20^\circ, |\vec{F_1}|\left(\tfrac{\sqrt{2}}{2}\right) + |\vec{F_2}|\sin 20^\circ\right)} = \overrightarrow{(0,100)},$$

then
$$\left.\begin{aligned} |\vec{F_1}|\left(-\tfrac{\sqrt{2}}{2}\right) + |\vec{F_2}|\cos 20^\circ &= 0 \\ |\vec{F_1}|\left(\tfrac{\sqrt{2}}{2}\right) + |\vec{F_2}|\sin 20^\circ &= 100 \end{aligned}\right\} \text{(add equations)}$$

$$\rightarrow \quad |\vec{F_2}|\cos 20^\circ + |\vec{F_2}|\sin 20^\circ = 100$$
$$\rightarrow \quad |\vec{F_2}|(\cos 20^\circ + \sin 20^\circ) = 100$$
$$\rightarrow \quad |\vec{F_2}| = \frac{100}{\cos 20^\circ + \sin 20^\circ} \approx \boxed{78.02\ N.};$$

$$|\vec{F_1}|\left(-\tfrac{\sqrt{2}}{2}\right) + |\vec{F_2}|\cos 20^\circ = 0 \rightarrow$$
$$|\vec{F_1}|\left(-\tfrac{\sqrt{2}}{2}\right) + (78.02)\cos 20^\circ \approx 0 \rightarrow$$
$$|\vec{F_1}|\left(\tfrac{\sqrt{2}}{2}\right) \approx (78.02)\cos 20^\circ \rightarrow$$
$$|\vec{F_1}| \approx \sqrt{2}\,(78.02)\cos 20^\circ \approx \boxed{103.68\ N};$$

The two force vectors can now be written as

$$\vec{F_1} = \overrightarrow{\left(|F_1|\left(-\frac{\sqrt{2}}{2}\right), |F_1|\left(\frac{\sqrt{2}}{2}\right)\right)}$$

$$\approx \overrightarrow{\left((103.68)\left(-\frac{\sqrt{2}}{2}\right), (103.68)\left(\frac{\sqrt{2}}{2}\right)\right)}$$

$$\approx \overrightarrow{(-73.31, 73.31)},$$

$$\vec{F_2} = \overrightarrow{(|F_2| \cos 20^\circ, |F_2| \sin 20^\circ)}$$

$$\approx \overrightarrow{((78.02)(0.9397), (78.02)(0.3420))}$$

$$\approx \overrightarrow{(73.32, 26.68)}$$

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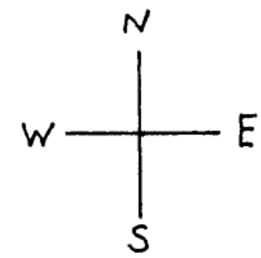
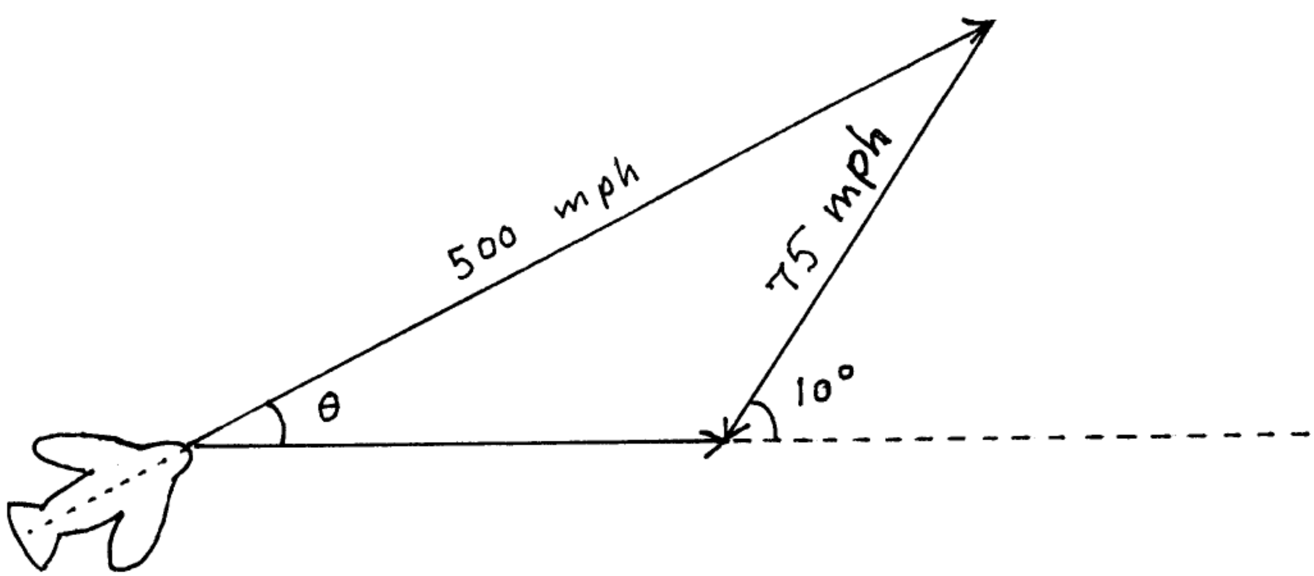
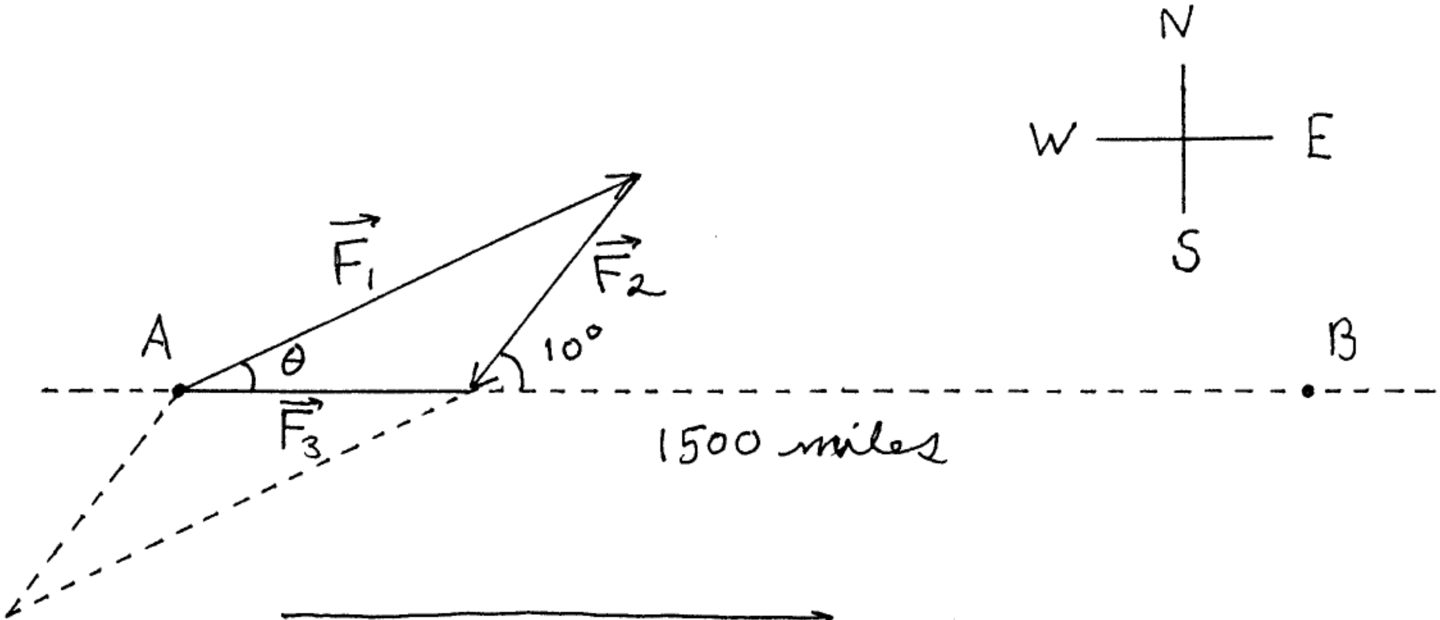


Example : A jet airplane wants to fly in a straight line from airport A directly East to airport B, which is 1500 miles away. The jet faces a headwind from 10° North of East at 75 mph. If the jet flies at a constant speed of 500 mph (relative to the surrounding air)

a.) in what direction should the jet fly ?
b.) what is the jet's actual flying speed (relative to the ground) ?
c.) how long will the flight take ?





$$\vec{F_1} = \overrightarrow{(|\vec{F_1}|\cos\theta,\ |\vec{F_1}|\sin\theta)}$$

$$= \overrightarrow{(500\cos\theta,\ 500\sin\theta)}\ ,$$

$$\vec{F_2} = -\overrightarrow{(75\cos 10°,\ 75\sin 10°)}$$

$$= \overrightarrow{(-75\cos 10°,\ -75\sin 10°)}\ ,$$

$$\vec{F_3} = \overrightarrow{(|\vec{F_3}|\cos 0°,\ |\vec{F_3}|\sin 0°)}$$

$$= \overrightarrow{(|\vec{F_3}|,\ 0)}\ ; \quad \text{then}$$

$$\vec{F_1} + \vec{F_2} = \vec{F_3}\ , \quad \text{so that}$$

$$\overrightarrow{(500\cos\theta - 75\cos 10°,\ 500\sin\theta - 75\sin 10°)} = \overrightarrow{(|\vec{F_3}|, 0)}$$

$$\longrightarrow \left.\begin{array}{l} 500\cos\theta - 75\cos 10° = |\vec{F_3}| \\ 500\sin\theta - 75\sin 10° = 0 \end{array}\right\}$$

$$\longrightarrow \quad 500\sin\theta = 75\sin 10°$$

$$\longrightarrow \quad \sin\theta = \frac{75\sin 10°}{500}$$

$\rightarrow \quad \theta = \arcsin\left(\frac{75 \sin 10^\circ}{500}\right)$

$\rightarrow \quad \boxed{\theta \approx 1.4926^\circ}$; then

$500 \cos\theta - 75 \cos 10^\circ = |\vec{F}_3|$

$\rightarrow \quad |\vec{F}_3| \approx 500 \cos(1.4926) - 75 \cos 10^\circ$

$\rightarrow \quad \boxed{|\vec{F}_3| \approx 426 \text{ mph}}$; and

$D = RT \rightarrow 1500 = 426T$

$\rightarrow \quad T = \frac{1500}{426}$

$\rightarrow \quad \boxed{T \approx 3.52 \text{ hrs.}}$