4(i) 5 pts

- correct proof:

  \[ \text{Proof.} \text{ Fix } r \in \mathbb{R} \text{ with } r \neq 0 \text{ and } r \neq 1, \text{ the proof proceeds by induction on } n. \]

  (i) if \( n = 1 \), then \( \sum_{i=0}^{1-1} ar^i = ar^0 = a = \frac{a(r^0 - 1)}{r - 1} \). So the base case holds.

  (ii) Suppose the statement is true for some natural number \( n \), we want to show the statement holds when \( n + 1 \). We have

  \[
  \sum_{i=0}^{n+1} ar^i = ar^{n+1} + \sum_{i=0}^{n} ar^i \\
  = ar^{n+1} + \frac{a(r^{n+1} - 1)}{r - 1} \\
  = \frac{ar^{n+1}(r - 1)}{r - 1} + \frac{a(r^{n+1} - 1)}{r - 1} \\
  = \frac{a(r^{n+2} + r^{n+1} + r^n - 1)}{r - 1} \\
  = \frac{a(r^{n+2} - 1)}{r - 1}
  \]

  By the PMI, the statement is true for all natural numbers \( n \). \( \square \)

- grading: 1 pt for the set-up (i.e. fix \( r \in \mathbb{R} \) with \( r \neq 0 \) and \( r \neq 1 \) & induction on \( n \)); 1 pt for showing the base case holds; 3 pts for showing the inductive step holds (i.e. 1 pt for assuming the statement holds for \( n \) and 2 pts for proving the equality above). Some might assume the statement holds true for \( n - 1 \) and prove the case \( n \), that is fine.

12(e) 2 pts

- correct answer: F. Justification: the statement is incorrect. The correct statement should be “for every natural number \( n \), \( n^2 + n \) is even”. As for the proof, in order to show that the base case is true, we need to prove that for \( n = 1 \), \( n^2 + n \) satisfies the property instead of \( n \) itself. The inductive step is fine (for the correct statement) as long as we assume \( n^2 + n \) is even instead of odd.

- grading: 1 pt for correct grade and 1 pt for reasonable justification (i.e. suffices to just explain why the claim is incorrect)