proof. Let $a$ be an integer and $p, q$ be distinct primes. Fsc assume $p|a$, $q|a$, but $pq \not|a$. Because $p|a$, $a = pm$ for some integer $m$. We claim that $q \not|m$. Indeed, if that is not the case, then $q|m$ and hence $pq|pm = a$, contradicting our assumption. We now have $q \not|m$, and because $q$ is prime, its only possible divisor is 1 and $q$, so $\gcd(q, m) = 1$. Together with $\gcd(q, pm) = \gcd(q, a) = q$, we obtain $\gcd(q, p) = q$ or otherwise $\gcd(q, a) < q$. But this again contradicts our hypothesis that $p, q$ are distinct primes, completing the proof.

grading: 5 pts if the proof is complete and correct (there should be more than one way to prove the statement); for every step that is unclear or not fully justified, -1 pt. Students can submit regrade requests if they have further questions.

Remark: The statement (“$p|a, q|a \Rightarrow pq|a$”) only holds when $p, q$ are primes, so if the student proves the statement without using this fact, something is wrong. For example: some might say $p \not|m \Rightarrow \gcd(p, m) = 1$. This is not true unless $p$ is prime (also covered in quiz 4), so they have to at least mention $p$ is prime to so the deduction is valid.

Section 1.8 21(e) 2 pts

− correct answer: F (failure). The statement is incorrect. Consider the counterexample where $a = 4$ and $b = 6$.
− grading: 1 pt for correct grading and 1 pt for reasonable explanation (counterexample is enough).
− Remark: for problems like this one, it is always helpful to check the statement with some simple examples (i.e. some prime numbers or some integers with common divisors).