MAT 108 Homework 3 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 1.3 #1fg, 4, 7, 10ab

1. (f) Let $A(x)$ be the statement "x is honest". The sentence is then $(\forall x)A(x) \vee \sim (\exists x)A(x)$

(g) Let $A(x)$ be the statement "x is honest". The sentence is then $(\exists x)A(x) \wedge (\exists y)A(y)$

4. It's not quite clear which five important properties in the appendix the problem is referring to, but we give some of them:

- For all x in $\mathbb{Z}, x < 0, x = 0$, or $x > 0$ can be written as
$$(\forall x \in \mathbb{Z})[x < 0 \vee x = 0 \vee x > 0].$$
- For all x in $\mathbb{Z}$, $x + 0 = x, x + (-x) = 0$, and $x \cdot 0 = 0$ can be written as
$$(\forall x \in \mathbb{Z})[x + 0 = x \wedge x + (-x) = 0 \wedge x \cdot 0 = 0].$$
- For all x in $\mathbb{Z}$, there is y in $\mathbb{Z}$ such that $x + y = 0$. can be written as
$$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})[x + y = 0].$$
- For all x, y, z in $\mathbb{Z}$, if $x < y$ and z is positive, $xz < yz$ can be written as
$$(\forall x, y, z \in \mathbb{Z})[(x < y \wedge z > 0) \implies xz < yz].$$
- For all x, y, z in $\mathbb{Z}$, if $x < y$ and z is negative, $xz < yz$ can be written as
$$(\forall x, y, z \in \mathbb{Z})[(x < y \wedge z > 0) \implies xz > yz].$$
- For all x and y in $\mathbb{Z}$, if x and y are both positive or both negative, then xy is positive can be written as
$$(\forall x, y \in \mathbb{Z})[(x > 0 \wedge y > 0) \vee (x < 0 \wedge y < 0) \implies xy > 0].$$
- For all x, y in $\mathbb{Z}$, if one of x or y is positive and the other negative, then xy is negative can be written as
$$(\forall x, y \in \mathbb{Z})[(x < 0 \wedge y > 0) \vee (x > 0 \wedge y < 0) \implies (xy < 0)].$$

7. (a) Let U be any universe.

$$\begin{aligned} \text{The sentence } \sim (\exists x)A(x) \text{is true in } U &\text{ iff } (\exists x)A(x) \text{ is false in } U \\ &\text{ iff the truth set of } A(x) \text{ is empty in the universe} \\ &\text{ iff the truth set of } \sim A(x) \text{ is the universe } U \\ &\text{ iff } (\forall x) \sim A(x). \end{aligned}$$

(b)

$$\begin{aligned} \sim (\exists x)A(x) &\text{ is equivalent to } \sim (\exists x) \sim\sim A(x) \\ &\text{ is equivalent to } \sim\sim (\forall x) \sim A(x) \\ &\text{ is equivalent to } (\forall x) \sim A(x) \end{aligned}$$

10. (a) True (additive inverses exist for real numbers)

(b) False. (additive inverses for real numbers are unique.)