MAT 108 Homework 3 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 1.6 #2d, 4d, 6d, 9efg

2. Prove for all integers a, b, and c

(d) If there exist integers m and n such that $am + bn = 1$ and $c \neq \pm 1$, then c does not divide a or c does not divide b.

Solution: Let a, b, and c be integers and suppose that there exist integers $m, n \in \mathbb{Z}$ such that $am + bn = 1$. Assume towards a contradiction that $c \neq \pm 1$ and c divides both a and b. By definition, we can then write $a = sc$ and $b = tc$ for some integers $s, t \in \mathbb{Z}$. Then, since $am+bn = 1$, we can substitute to get $msc+ntc = 1$, or $c(ms+nt) = 1$. Since $m, s, n, t \in \mathbb{Z}$, the number $ms+nt$ is also an integer, and therefore $1/c = ms + nt$ must be an integer as well. But $c \neq \pm 1$, so $1/c$ is not an integer. This is a contradiction because c cannot both be an integer and not be an integer. Thus, if there exist $m, n \in \mathbb{Z}$ such that $am + bn = 1$, then there does not exist a $c \neq \pm 1$ dividing both a and b.

4. Find a proof or counterexample

(d) For integers a, b, c, if a divides bc, then either a divides b or a divides c.

Solution: $a = 15, b = 9, c = 5$ is a counterexample.

6. (d) Prove that there is a natural number M such that for every natural number n, $1/n < M$.

Solution: Let $M = 2$. Then the inequality $1/n < 2$ is equivalent to $1 < 2n$. Since n is a natural number, we have $1 \leq n$. By properties of inequalities $1 \leq n$ and $1 < 2$ together imply $1 < 2n$. Thus, there exists M such that $1/n < M$ for all $n \in \mathbb{N}$.

9. 'Grade' the following proofs:

(e) (see textbook for proof)

Solution: F. The attempted proof only gives example for two of the cases, which does not constitute a proof of the claim, as we need to show that the claim holds for all real numbers. It's also possible to argue that this proof is worth a C based on the the solid structure and the fact that it could easily be made into a real proof.

(f) (see textbook for proof)

Solution: C. The proof implicitly relies on several facts that we've already proven. This means that it's a valid proof and mostly readable, but it would still be more complete if the writer stated that any prime number greater than 2 is odd, the sum of two odd numbers is even, and any even number greater than 2 is composite.

(g) (see textbook for proof)

Solution: A. Good proof by contradiction.