MAT 108 Homework 10 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 2.1 #6ef, 11c, 12, 13, 19bdf

6. Give an example, if there is one, of sets A, B, and C such that the following are true. If there is no example, write "Not possible."

(e) $A \in B$, $A \subseteq B$, and $B \subseteq C$.

Solution: One example might be $A = \{x\}$, $B = \{\{x\}, x\}$ and $C = \{\{x\}, x, y\}$.

(f) $A \subseteq B$, $A \not\subseteq C$, and $B \subseteq C$.

Solution: Not possible.

11c. Prove that $\{x \in \mathbb{R} : 2|x+3| + x = 0\} = \{-6, -2\}$. **Solution:** Let $A = \{x \in \mathbb{R} : 2|x+3| + x = 0\}$ and

$B = \{-6, -2\}$. Then $B \subseteq A$, since $2|-6+3| - 6 = 6 - 6 = 0$ and $2|-2+3| - 2 = 2 - 2 = 0$.
To show that $A \subseteq B$, we consider the cases where (i) $x + 3 \geq 0$ and (ii)$x + 3 < 0$.

(i) First, suppose $x + 3 \geq 0$. Then

$$2|x+3| + x = 2(x+3) + x = 2x + 6 + x = 3x + 6.$$

Setting this expression equal to 0 and solving then yields $x = -2$. Therefore, $-2 \in A$.

(ii) Now, suppose $x + 3 < 0$. Then

$$2|x+3| + x = -2(x+3) + x = -x - 6.$$

Setting this expression equal to 0 and solving yields $x = 6$. Therefore, $-6 \in A$ and since for all $x \in \mathbb{R}$, x satisfies either $x + 3 \geq 0$ or $x + 3 < 0$, there are no other solutions to the equation $2|x+3| + x = 0$. Thus, $A \subseteq B$. Since we have $A \subseteq B$ and $B \subseteq A$, we have shown $A = B$.

12. Prove that there is only one empty set. That is, prove that if A and B are sets with no elements, then $A = B$.

Solution: let A and B be two sets with no elements. Since A has no elements and we know from lecture that a set with no elements is contained in every other set, we have $A \subseteq B$. By the exact same reasoning $B \subseteq A$. But this can only happen when $A = B$. Thus, the empty set is unique.

13. For a natural number a, let $a\mathbb{Z}$ be the set of all integer multiples of a. Prove that for all $a, b \in \mathbb{N}$, $a = b$ if and only if $a\mathbb{Z} = b\mathbb{Z}$.

Solution: Ask for solution on Piazza or in James's office hours.

19. 'Grade' the following proofs:

(b) (see textbook for proof)

Solution: F. Attempted proof is an example.

(d) (see textbook for proof)

Solution: F. Attempted proof does not use the correct set containments to show that any element of A is contained in C. A case could be made for C, since the proof would be mostly correct if you reversed all of the set containments.

(f) (see textbook for proof)

Solution: F. The claim is false and the attempted proof only appears to prove the claim because their casework fails to consider $-4 < x < 0$. Structure of the argument is solid otherwise.