MAT 108 Homework 11 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 2.2 #1j, 2i, 9e, 10d, 11df, 20abc

1. Let $A = \{1, 3, 5, 7, 9\}$, $B = \{0, 2, 4, 6, 8\}$, $C = \{1, 2, 4, 5, 7, 8, \}$ and $D = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$. Find

(j) $(A \cup B) - (C \cap D)$. **Solution:** Given A, B, C, and D as above, we have $A \cup B$ is the set of integers between 0 and 9, while $C \cap D = \{1, 2, 5, 7, 8\}$. Then $(A \cup B) - (C \cap D) = \{0, 3, 4, 6, 9\}$.

2. Let the universe be all real numbers. Let $A = [3, 8)$, $B = [2, 6]$, $C = (1, 4)$, and $D = (5, \infty)$. Find

(i) $(A \cup C) - (B \cap D)$ **Solution:** Given A, B, C, and D as above, we have $A \cup B = [2, 8)$, while $C \cap D = \emptyset$. Then $(A \cup B) - (C \cap D) = A \cup B = [2, 8)$.

9. Let A, B, and C be sets. Prove that

(e) (A-B)-C=(A-C)-(B-C). **Solution:** Let A, B, C be sets and first consider $x \in (A - B) - C$. Then $x \in A - B$ and $x \notin C$. Since $x \in A - B$, we know that $x \in A$ and $x \notin B$. Therefore, $x \in A - C$ and $x \notin B - C$. Therefore, $x \in (A - C) - (B - C)$, so $(A - B) - C \subseteq (A - C) - (B - C)$.

Now, consider $x \in (A - C) - (B - C)$. Then $x \in (A - C)$ and $x \notin (B - C)$. Therefore, $x \in A$ and $x \notin C$. Since $x \notin B - C$ and $x \notin C$, we can conclude that $x \notin B$. It follows that $x \in A - B$. Then $x \notin C$, and $x \in A - B$ implies $x \in (A - B) - C$, so $(A - C) - (B - C) \subseteq (A - B) - C$. Thus, since we have shown both sets contain each other, $(A - B) - C = (A - C) - (B - C)$.

10. Let A, B, C, and D be sets. Prove that

(d) if $C \subseteq A$ and $D \subseteq B$, then $D - A \subseteq B - C$. **Solution:**
Ask for solution on Piazza or in James's office hours.

11. Provide counterexamples for each of the following.

(d) $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$. **Solution:** Let $A = \{x, y\}$ and $B = \{y, z\}$. Then $\mathcal{P}(A) = \{\{x\}, \{y\}, \{x, y\}\}$ and $\mathcal{P}(B) = \{\{y\}, \{z\}, \{y, z\}\}$, so $\mathcal{P}(A) - \mathcal{P}(B) = \{\{x\}, \{x, y\}\}$. But $A - B = x$, so $\mathcal{P}(A - B) = \{\{x\}\}$, which does not contain $\{\{x\}, \{x, y\}\}$.

(f) A-(B-C)=(A-B)-C. **Solution:** Let $A = \{x, y, z\}$, $B = \{y, z\}$ and $C = \{x, y\}$. Then $A - (B - C) = A - \{z\} = \{x, y\}$ while $(A - B) - C = \{x\} - C = \emptyset$.

20. 'Grade' the following proofs:

(a) (see textbook for proof)

Solution: F. It's not quite clear why x would not be in some arbitrary set C. The attempted proof does not clearly show the implication in the claim.

(b) (see textbook for proof)

Solution: C. The attempted proof is a little hard to read, but if you replace the statement 'Suppose $A - C$' with 'Suppose $x \in A - C$' and similarly for the $B - C$ later in the proof, the argue is mostly complete.

(c) (see textbook for proof)

Solution: C. The attempted proof fails to define x in the beginning and is a bit fast and loose with where x lives throughout. The argument is about as difficult to follow as the attempted proof in part b, but is mostly there.