MAT 108 Homework 16 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 3.1 #7adg, 10abc, 11bc, 12b, 17abc

7. Let $R = \{(1,5),(2,2),(3,4),(5,2)\}$, $S = \{(2,4),(3,4),(3,1),(5,5)\}$, and $T = \{(1,4),(3,5),(4,1)\}$. Find

(a) $R \circ S$.

Solution: $R \circ S = \{(3,5),(5,2)\}$

(d) $R \circ R$.

Solution: $R \circ R = \{(1,2),(2,2),(5,2)\}$

(g) $R \circ (S \circ T)$.

Solution: $R \circ (S \circ T) = \{(3,2)\}$

10. Let $A = \{a,b,c,d\}$. Give an example of relations R, S and T on A such that

(a) $R \circ S \neq S \circ R$

Solution: (Answers may vary) Take $R = \{((a,b))\}$ and $S = \{(b,a)\}$. Then $R \circ S = \{(b,b)\}$ and $S \circ R = \{(a,a)\}$.

(b) $(S \circ R)^{-1} \neq S^{-1} \circ R^{-1}$.

Solution: Take $R = \{((a,b))\}$ and $S = \{(b,a)\}$. Then $R^{-1} = \{(b,a)\}, S^{-1} = \{(a,b)\}, (S \circ R)^{-1} = \{((a,a)\}$, and $S^{-1} \circ R^{-1} = \{(b,b)\}$.

(c) $S \circ R = T \circ R$, but $S \neq T$.

Solution: Take $R = \{((a,b))\}, S = \{(b,a)\}$, and $T = \{(b,a),(c,d)\}$. Then we have $S \circ R = T \circ R = \{(b,b)\}$, but $S \neq T$.

11. Let R be a relation from A to B and S be a relation from B to C.

(b) Prove that $\mathrm{Dom}(S \circ R) \subseteq \mathrm{Dom}(R)$.

Solution: Let A, B, C, R and S be given as in the statement of the problem and suppose we have some $a \in \mathrm{Dom}(S \circ R)$. By definition, there exists some $c \in C$ such that $(a,c) \in S \circ R$. Therefore, there exists some $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$. Since $(a,b) \in R$, we have that $a \in \mathrm{Dom}(R)$, again by definition. Thus, $\mathrm{Dom}(S \circ R) \subseteq \mathrm{Dom}(R)$.

(c) Show by example that $\mathrm{Dom}(S \circ R) = \mathrm{Dom}(R)$ may be false.

Solution: (Answers may vary) Take $A = B = C = \{1,2,3\}$ and define the relations $R = \{(1,2),(2,3)\}$ and $S = \{(2,1)\}$. Then $\mathrm{Dom}(S \circ R) = \{1\}$, while $\mathrm{Dom}(R) = \{1,2\}$

12. Complete the proof of Theorem 3.1.2 by proving that if R is a relation from A to B and S is a relation from B to C, then

(b) $R \circ I_A = R$.

Solution: Ask for solution on Piazza or in James's office hours.

17. 'Grade' the following proofs:

(a) (see textbook for proof)
Solution: F. The claim is false and the first if and only if of the attempted proof is false.

(b) (see textbook for proof)
Solution: F. The claim is false and there the attempted proof assumes the existence of the relation $(x, y) \in R$ when it's possible that no such relation exists.

(c) (see textbook for proof)
Solution: F. The overall claim is false and the claim in the proof that $x = y$ is not true in general.