**MAT 108 Homework 17 Solutions**

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 3.2 #1bgi, 2g, 3g, 4a, 8bc, 19abc

**1**. Indicate which of the following relations on the given sets are reflexive on the given set, which are symmetric, and which are transitive.

(b) $\leq$ on $\mathbb{N}$.

**Solution:** Reflexive ($x \leq x$), Transitive ($x \leq y, y \leq z \implies x \leq z$), not symmetric ($x \leq y \not\Longrightarrow y \leq x$ in general)

(g) "divides" on $\mathbb{N}$

**Solution:** Reflexive ($x|x$), Transitive ($x|y, y|z \implies x|z$), not symmetric ($x|y \not\Longrightarrow y|x$ in general)

(i) {(1,5), (5,1), (1,1)} on the set $A = \{1, 2, 3, 4, 5\}$

**Solution:** Not reflexive ($(2, 2) \notin A$), Symmetric, not transitive ((5,1) and (1,5) in $A$ but (5,5) not in $A$)

**2**. Let $A = \{1, 2, 3\}$. List the ordered pairs, and draw the digraph of a relation on $A$ with the given properties.

(g) Not reflexive, symmetric, and transitive

**Solution:** (Answers may vary) $R = \{(1, 2), (2, 1), (2, 2), (1, 1)\}$. Not reflexive because $(3, 3)$ not in $R$.
Ask in OHs or on Piazza for digraph.

**3**. For each part of Exercise 2, give an example of a relation on $\mathbb{R}$ with the desired properties.

(g) Not reflexive, symmetric, and transitive

**Solution:** $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | xy > 0\}$. $R$ is not reflexive because $(0, 0) \notin R$, $R$ is symmetric because $xy = yx$, so $(x, y) \in R$ implies $(x, y) \in R$. Finally, $R$ is transitive because $xy > 0$ and $yz > 0$ implies $xz > 0$.
Quick justification for the last fact: $xy > 0$ implies $x, y$ either both positive or both negative. Similarly for $y, z$. Therefore, $x$ and $z$ are either both positive or both negative, so $xz > 0$.

**4**. The properties of reflexivity, symmetry, and transitivity are related to the identity relation and the operations of inversion and composition. Prove that

(a) $R$ is a reflexive relation on $A$ if and only if $I_A \subseteq R$.

**Solution:** ($\implies$) Let $R$ be a reflexive relation on $A$ and let $(x, y)$ be an element in $I_A$. By the definition of the Identity relation, we must have $y = x$, so $(x, y) = (x, x)$. Moreover, since $R$ is reflexive and $x \in A$ we must also have $(x, x) \in R$. Therefore, $I_A \subseteq R$.

( $\Longleftarrow$ ) Now let $R$ be a relation on $A$ and assume $I_A \subseteq R$. By definition of the identity relation, we must have $(x, x) \in I_A$ for all $x \in A$. Since $I_A \subseteq R$, this implies $(x, x) \in R$ for all $x \in A$. This is precisely the definition of reflexivity, so therefore, $I_A \subseteq R$ implies $R$ is reflexive.

Since we have shown both implications, it follows that $R$ is reflexive if and only if $I_A \subseteq R$.

**8**. Which of the digraphs pictured in the textbook represent relations that are (i) reflexive on the given set (ii) symmetric? (iii) transitive?

(b) Reflexive, not transitive, not symmetric

**Solution:**

(c) Reflexive, transitive, and symmetric

**Solution:**

**19.** 'Grade' the following proofs:

(a) (see textbook for proof)
**Solution:** F. Claim is false. Does not show that $(x, x) \in R$ for *all* $x$.

(b) (see textbook for proof)
**Solution:** F. To show $T$ is symmetric, we need to show that if $(x, y)T(r, s)$, then $(r, s)T(x, y)$.

(c) (see textbook for proof)
**Solution:** A.