Math 127B (1.5)xPractice Final Spring 2025

You may use one sheet of notes.
The final will have only six problems.

1. Consider the zigzag function f defined on $[-1,1]$ with $f(0)=0$, for every positive integer n there is $f(\frac{\pm 1}{n}) = \frac{(-1)^n}{n^2}$ and f is linear on the intervals $[\frac{-1}{n}, \frac{-1}{n+1}]$ and , $[\frac{1}{n+1}, \frac{1}{n}]$.
   At which points in $(-1,1)$ is f differentiable?

2. Show that if f is thrice differentiable in (a,b) and $a<c<b$ then
$$f''(c) = \lim_{h\to 0} h^{-2}[f(c-h) - 2f(c) + f(c+h)].$$

3. Assume that f is twice differentiable at every point in $(0,4)$, $f(3)=3$, $f(1)=0$ and there is some other number $x \in (1,3)$ with $f(x)=0$ also.
   Show that there is some number $y \in (1,3)$ with $f''(y) > \frac{3}{4}$.

4. Consider the sequence of functions $\{f_n\}$ defined on $[0,\infty)$ with $f_n(x) = \frac{nx}{1+nx}$.
   (a) Find the pointwise limit of this sequence.
   (Pay attention to zero.)
   (b) Determine whether the sequence converges uniformly.

5. Consider the two equal parts partition $P = \{0, \frac{1}{2}, 1\}$ of $[0,1]$. Show that there is another two part partition Q of $[0,1]$ so that $U(x^2,Q) - L(x^2;Q) < U(x^2,P) - L(x^2;P)$.

6. Assume that f is integrable on $[0,4]$ and define $F(x) = \int_0^x f$ also on $[0,4]$.
   Show that there is some $c \in [2,3]$ with $\int_2^3 F = \int_0^c f$.

7. Show that if $\int_I f^2 = 0$ then $\int_I f$ exists.

8. Show that if $f(0) = 0$ and $f(x) = P_{\infty,0}(x)$ (its Taylor series) on $(-a, a)$ then so does $g(x)$ with $g(0) = f'(0)$ and $g(x) = x^{-1}f(x)$ otherwise.

9. Consider the function $f(x) = \int_0^x \ln(1+t)dt$.

   (a) Find the Taylor polynomial $P_{2,0}(x)$ for f.

   (b) Show that if $|x| < 1$ then $\frac{-x^3}{6} \geq f(x) - P_{2,0}(x) \geq \frac{-x^3}{6(x^2-2x+1)}$.