Math 127B Midterm II Spring 2025

You may use one sheet of notes.

Your score will be the best 4 of 5.

1. Find
$$\sum_{n=1}^{\infty} \frac{1}{n2^n}.$$

2. Find a function f with
$$U(f;\{0,1\}) - L(f;\{0,1\}) = 1$$
and
$$U\left(f;\left\{0,\frac{1}{2},1\right\}\right) - L\left(f;\left\{0,\frac{1}{2},1\right\}\right) = \frac{1}{2}.$$

3. Show that $\int_0^1 \chi_{\frac{1}{2}} = 0$ by finding for every $\epsilon > 0$ a partition P of $[0,1]$ with $-\epsilon < L(\chi_{\frac{1}{2}}; P) < U(\chi_{\frac{1}{2}}; P) < \epsilon$.
Here $\chi_{\frac{1}{2}}$ is the characteristic function defined by $\chi_{\frac{1}{2}}(\frac{1}{2}) = 1$ and if $x \in [0,1] - \{\frac{1}{2}\}$ then $\chi_{\frac{1}{2}}(x) = 0$. Your P will depend on ϵ.

4. Assume that f and g are bounded functions on $[0,1]$.
   (a) Show that if f and g are continuous at c then so is $f+g$.
   (b) Show that $D(f+g) \subseteq D(f) \cup D(g)$.

   Here $D(f)$ is the set of values in $[0,1]$ at which f is not continuous.

5. Assume that f is monotone increasing from 0 to 1 on $[0,1]$ (that is: if $0 \le x \le y \le 1$ then $0 = f(0) \le f(x) \le f(y) \le f(1) = 1$).
Show that $\lim_{n\to\infty} \int f^n$ exists.