The hypothesis in Theorem 6.3.1 is unnecessarily strong. We actually do not need to assume that \( f_n(x) \to f(x) \) at each point in the domain because the assumption that the sequence of derivatives \( (f'_n) \) converges uniformly is nearly strong enough to prove that \( (f_n) \) converges, uniformly in fact. Two functions with the same derivative may differ by a constant, so we must assume that there is at least one point \( x_0 \) where \( f_n(x_0) \to f(x_0) \).

**Theorem 6.3.2.** Let \( (f_n) \) be a sequence of differentiable functions defined on the closed interval \([a, b]\), and assume \( (f'_n) \) converges uniformly on \([a, b]\). If there exists a point \( x_0 \in [a, b] \) where \( f_n(x_0) \) is convergent, then \( (f_n) \) converges uniformly on \([a, b]\).

**Proof.** Exercise 6.3.5.

Combining the last two results produces a stronger version of Theorem 6.3.1.

**Theorem 6.3.3.** Let \( (f_n) \) be a sequence of differentiable functions defined on the closed interval \([a, b]\), and assume \( (f'_n) \) converges uniformly to a function \( g \) on \([a, b]\). If there exists a point \( x_0 \in [a, b] \) for which \( f_n(x_0) \) is convergent, then \( (f_n) \) converges uniformly. Moreover, the limit function \( f = \lim f_n \) is differentiable and satisfies \( f' = g \).