Math 127B 2020 HW24

The spaces \( IRInt(J) \) of improperly Riemann integrable functions on a possibly unbounded interval \( J \) are defined so that the analog of 11.44 holds: if \( J, K \) and \( J \cup K \) are intervals and \( f|_J \in IRInt(J) \) and \( f|_K \in IRInt(K) \) then \( f|_{J \cup K} \in IRInt(J \cup K) \).

1. Determine whether these spaces are linear. (analogous to 11.32 and 11.33)
2. Determine whether these spaces are uniformly closed. (analogous to 12.17)
3. Determine whether these spaces are closed under changes at finitely many points. (analogous to 11.46)
4. Determine whether these spaces are closed under pointwise products. (analogous to 11.35)
5. Determine whether these spaces are closed under pointwise multiplication by bounded continuous functions. (another analog to 11.35)

Hint for (2): Consider first showing that the sequence \( f_n(x) = x^{1-n^{-1}} \) converges uniformly in \([1, \infty)\) (though not in \((0,1]\)).