Fall 2018 Problem 3: Show that for every $f \in C(\mathbb{T})$ and $\epsilon > 0$ there is an initial condition $g \in C(\mathbb{T})$ for which there is a solution $u(x,t)$ to the heat equation on a ring with $u(x,0) = g(x)$ and $|u(x,1) - f(x)| < \epsilon$ for every $x \in \mathbb{T}$.

Fall 2014 Problem 6:

(a) By choosing a suitable even, periodic extension for f, calculate the Fourier cosine series for $f = \sin x$, $x \in [0, \pi]$.

(b) Deduce that

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = 1/2.$$

Spring 2016 Problem 4: Suppose f is a function in the Schwartz space $S(\mathbb{R})$ which satisfies the normalizing condition $\int_{-\infty}^{\infty} |f(x)|^2 dx = 1$. Let $\hat{f}$ denote the Fourier transform of f. Show that

$$\left(\int_{-\infty}^{\infty} x^2 |f(x)|^2 dx \right) \left(\int_{-\infty}^{\infty} \omega^2 |\hat{f}(\omega)|^2 d\omega \right) \geq \frac{1}{16\pi^2}.$$

Fall 2018 Problem 4: Consider the functions $f_N(x) = (2\pi)^{-1} \sum_{n=-N}^{N} e^{inx}$. Show that if $g \in L^2(\mathbb{T})$ then $\{f_N * g\}$ converges in $\|\cdot\|_{L^2}$-norm to g (here, $*$ denotes convolution).

Fall 2016 Problem 2: Let $f : \mathbb{T} \to \mathbb{C}$ be a C^1 function such that $\int_{-\pi}^{\pi} f(x) dx = 0$. Show that

$$\int_{-\pi}^{\pi} |f(x)|^2 dx \leq \int_{-\pi}^{\pi} |f'(x)|^2 dx.$$