Fall 2014 Problem 2: Define the bounded linear operator $K : L^2(0,1) \to L^2(0,1)$ by

$$(Kf)(x) = \int_0^1 xy(1-xy)f(y)dy.$$

Find the spectrum or K and classify it.

Spring 2016 Problem 2: Let M be a multiplication on $L^2(\mathbb{R})$ defined by

$$Mf(x) = m(x)f(x),$$

where $m(x)$ is continuous and bounded. Prove that M is a bounded operator on $L^2(\mathbb{R})$ and that the spectrum is given by

$$\{m(x) : x \in \mathbb{R}\}^{cl},$$

where A^{cl} denotes the closure of A. Can M have eigenvalues?

Fall 2003 Problem 5 and Winter 2005 Problem 4: Find the point spectra of R and L the right and left shift operators on $\ell^2(\mathbb{N})$. (That is $R(x_1, x_2, x_3, \ldots) = (0, x_1, x_2, \ldots)$ and $L(x_1, x_2, x_3, \ldots) = (x_2, x_3, x_4, \ldots)$).

Fall 2012 Problem 3: Let T be a bounded linear operator on a Hilbert space H. Show that

(a) If $\|T\| \leq 1$, then T and its adjoint operator T^* have the same fixed point. i.e. Show that for $x \in H$,

$$Tx = x \Longleftrightarrow T^*x = x.$$

(b) Let λ be an eigenvalue of T. Is it true that its complex conjugate $\overline{\lambda}$ must be an eigenvalue of T^*? Is it true that $\overline{\lambda}$ must be in the spectrum of T^*? Justify your answers.

Winter 2007 Problem 6: Show that the spectrum of every unitary operator lies on the unit circle.

Winter 2008 Problem 4 Assume that H is a complex seperable Hilbert space and $A \in B(X)$ with spectrum σ, resolvent set ρ and resolvent operators $R(\mu) = (\mu I - A)^{-1}$.

(1) * Show that if $\mu \in \rho$ and $|\nu - \mu| < \|R(\mu)\|^{-1}$ then $R(\nu) = [I - (\mu - \nu)R(\mu)]^{-1}R(\mu)$.

(2) Show that if $\mu \in \rho$ then $\|R(\mu)\| \geq (d(\mu, \sigma))^{-1}$.

Fall 2012 Problem 2: Assume that $k \in C_{\mathbb{R}}[0,1]$ and define $T \in B(L^2[0,1])$ by setting $(Tf)(x) = k(x)\int_0^1 k(t)f(t)dt$ for any $f \in L^2[0,1]$.

(1) Show that T is self-adjoint.

(2) Show that $T^2 = rT$ for some number r.

(3) Find the spectral radius of T.

Spring 2013 Problem 2: Show that if $\{T_i\} \in B(X)$ is a sequence of bounded operators on a Banach space X which all have the same spectrum $\sigma = \sigma(T_i)$ and converge in norm to T then $\sigma \subseteq \sigma(T)$.

Fall 2013 Problem 2: Define $T : L^2[0,1] \to L^2[0,1]$ by $(Tf)(x) = \int_0^x f(t)dt$ for $f \in L^2[0,1]$ and $x \in [0,1]$.

(1) * Show that T is bounded.

(2) * Show that T has no eigenvalues.
(3) Find $\lim_{n\to\infty} \|T^n\|$.