Please write up one of the following problems previously discussed in class. We will discuss how you approached the presentation tomorrow.

Spring 2014 Problem 4:

Let P_1 and P_2 be a pair of orthogonal projections onto H_1 and H_2, respectively, where H_1 and H_2 are closed subspaces of a Hilbert space H. Prove that P_1P_2 is an orthogonal projection if and only if P_1 and P_2 commute. In that case, prove that P_1P_2 is the orthogonal projection onto $H_1 \cap H_2$.

(Spring 2015 Problem 4) Let $f_n : [0,1] \to \mathbb{R}$ be a sequence of measurable functions. Suppose

(i) $\int_0^1 |f_n(x)|^2 dx \leq 1$ for $n = 1, 2, \cdots$.

(ii) $f_n \to 0$ almost everywhere.

Show that

$$\lim_{n\to\infty} \int_0^1 f_n(x)dx = 0.$$

Spring 2016 Problem 4: Suppose f is a function in the Schwartz space $S(\mathbb{R})$ which satisfies the normalizing condition $\int_{-\infty}^{\infty} |f(x)|^2 dx = 1$. Let $\hat{f}$ denote the Fourier transform of f. Show that

$$\left(\int_{-\infty}^{\infty} x^2 |f(x)|^2 dx\right) \left(\int_{-\infty}^{\infty} \omega^2 |\hat{f}(\omega)|^2 d\omega\right) \geq \frac{1}{16\pi^2}.$$

Fall 2003 Problem 5 and Winter 2005 Problem 4: Find the point spectra of R and L the right and left shift operators on $\ell^2(\mathbb{N})$. (That is $R(x_1, x_2, x_3, \ldots) = (0, x_1, x_2, \ldots)$ and $L(x_1, x_2, x_3, \ldots) = (x_2, x_3, x_4, \ldots)$).