Spring 2010 Problem 1 Let (X, d) be a complete metric space, $\overline{x} \in X$ and $r > 0$. Set $D := \{x \in X : d(x, \overline{x}) \leq r\}$, and let $f : D \to X$ satisfying

$$d(f(x), f(y)) \leq kd(x, y)$$

for any $x, y \in D$, where $k \in (0, 1)$ is a constant.
Prove that if $d(\overline{x}, f(\overline{x}) \leq r(1 - k)$ then f admits a unique fixed point.

Spring 2010 Problem 6 Let $f : [0,1] \to \mathbb{R}$. Show that f is continuous if and only if the graph of f is compact in $\mathbb{R}^2$.

Fall 2011 Problem 1 Let (X, d) be a metric space and let (x_n) be a sequence in X. Call $x \in X$ a *cluster point* o (x_n) iff there is a subsequence $(x_{n_k})_{k \leq 0}$ with limit x.

(1) Let (a_n) be a sequence of distinct points in X. Construct a sequence in X for which every a_k is a cluster point.
(2) Can a sequence in a metric space have an uncountable number of cluster points? Prove your answer.

Fall 2017 Problem 1 Prove that every metric subspace of a separable metric space is separable.

Winter 2008 Problem 2: Consider $X = \mathbb{R}^2$ with the Euclidean metric e. Define $d : X \times X \to \mathbb{R}$ by $d(x, y) = e(x, y)$ if x and y lie on the sae ray through te origin and $d(x, y) = e(x, 0) + e(0, y)$ otherwise.

(1) Prove that (X, d) is a metric space.
(2) Give an example of a set which is open in (X, d) but not in (X, e).