Math 21B Practice Final Spring 2025

You may use one page of notes but not a calculator or textbook.
Please do not simplify your answers.

Basic and Trigonometric Integrals

$\int x^n dx$	$=$	$\frac{1}{n+1}x^{n+1} + C$
$\int x^{-1} dx$	$=$	$\ln\lvert x\rvert + C$
$\int e^x dx$	$=$	$e^x + C$
$\int \sin(x) dx$	$=$	$-\cos(x) + C$
$\int \cos(x) dx$	$=$	$\sin(x) + C$
$\int \frac{dx}{\sqrt{1-x^2}} dx$	$=$	$\arcsin(x) + C = -\arccos(x) + C$
$\int \sec^2(x) dx$	$=$	$\tan(x) + C$
$\int \tan(x) dx$	$=$	$\ln\lvert\sec(x)\rvert + C$
$\int \csc^2(x) dx$	$=$	$-\cot(x) + C$
$\int \cot(x) dx$	$=$	$\ln\lvert\sin(x)\rvert + C$
$\int \frac{dx}{1+x^2} dx$	$=$	$\arctan(x) + C = -\text{arccot}(x) + C$
$\int \sec(x)\tan(x) dx$	$=$	$\sec(x) + C$
$\int \sec(x) dx$	$=$	$\ln\lvert\sec(x) + \tan(x)\rvert + C$
$\int \csc(x)\cot(x) dx$	$=$	$-\csc(x) + C$
$\int \csc(x) dx$	$=$	$-\ln\lvert\csc(x) + \cot(x)\rvert + C$
$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} dx$	$=$	$\text{arcsec}(x) + C = -\text{arccsc}(x) + C$

1. A function has the following values:

x	0	2	4	6	8	10	12
$f(x)$	4	3	5	6	7	6	5

   Estimate the integral $\int_0^{12} f(x)dx$ using three equal length intervals and the midpoint rule.

2. Find the number:
$$\int_{-1}^{1}(x+1)e^{x+1}dx.$$

3. Find the number:
$$\int_0^2 \frac{dx}{9-x^2}.$$

4. Find the antiderivative with constant of integration:
$$\int \frac{dx}{9+x^2}.$$

5. Determine whether the improper integral converges:
$$\int_0^\infty \frac{dx}{9+x^2}.$$

6. Consider the curve $y=\sqrt{9+x^2}$ between $x=0$ and $x=4$. Write an integral for the length of the curve. You do not need to evaluate the integral.

7. Consider the solid obtained by revolving about the x-axis the region bounded above by the curve $y=\sqrt{9+x^2}$, below by the x-axis and between $x=0$ and $x=4$.
   Find the volume of the solid.

8. Find the work in Nm required to lift an anchor which weighs $100N$ along with its $10m$ chain which weighs $100\frac{N}{m}$.

The last three problems find the slopes of curves at the origin.

9. Find $f'(0)$ if $f(x) = \int_0^{3x} \cos(t^2)dt$.
   Hint: no integration is required.

10. Find the slope $\frac{dy}{dx}$ at the point where $t = 0$ of the curve given parametrically by $x = t + t^2$ and $y = te^t$.

11. Consider the curve given in polar coordinates by $r = \theta$.
    Graph the points where θ is $\frac{-\pi}{4}$, 0 and $\frac{\pi}{4}$ and
    find the slope $\frac{dy}{dx}$ at the point where θ is 0.