Math 21B Midterm II Spring 2025: Wed May 7 3:10-4:00

You may use one page of notes but not a calculator or textbook. There are tables of integrals and trigonometric identities on the next page. Please do not simplify your answers.

Name:

ID:

Basic and Trigonometric Integrals

$\int x^n dx$	$=$	$\frac{1}{n+1}x^{n+1} + C$
$\int x^{-1} dx$	$=$	$\ln\lvert x\rvert + C$
$\int e^x dx$	$=$	$e^x + C$
$\int \sin(x) dx$	$=$	$-\cos(x) + C$
$\int \cos(x) dx$	$=$	$\sin(x) + C$
$\int \frac{dx}{\sqrt{1-x^2}} dx$	$=$	$\arcsin(x) + C = -\arccos(x) + C$
$\int \sec^2(x) dx$	$=$	$\tan(x) + C$
$\int \tan(x) dx$	$=$	$\ln\lvert\sec(x)\rvert + C$
$\int \csc^2(x) dx$	$=$	$-\cot(x) + C$
$\int \cot(x) dx$	$=$	$\ln\lvert\sin(x)\rvert + C$
$\int \frac{dx}{1+x^2} dx$	$=$	$\arctan(x) + C = -\mathrm{arccot}(x) + C$
$\int \sec(x)\tan(x) dx$	$=$	$\sec(x) + C$
$\int \sec(x) dx$	$=$	$\ln\lvert\sec(x) + \tan(x)\rvert + C$
$\int \csc(x)\cot(x) dx$	$=$	$-\csc(x) + C$
$\int \csc(x) dx$	$=$	$-\ln\lvert\csc(x) + \cot(x)\rvert + C$
$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} dx$	$=$	$\mathrm{arcsec}(x) + C = -\mathrm{arccsc}(x) + C$

Trigonometric Identities

1	$=$	$\cos^2(x) + \sin^2(x)$
1	$=$	$\sec^2(x) - \tan^2(x)$
$\cos^2(x)$	$=$	$\frac{1}{2}[1 + \cos(2x)]$
$\sin^2(x)$	$=$	$\frac{1}{2}[1 - \cos(2x)]$
$\cos(a+b)$	$=$	$\cos(a)\cos(b) - \sin(a)\sin(b)$
$\sin(a+b)$	$=$	$\sin(a)\cos(b) + \cos(a)\sin(b)$

1. (50 points: Integration)

   (a) Find the antiderivative with constant of integration:

$$\int \sin(2x)e^{\cos(2x)}dx.$$

   (b) Find the antiderivative with constant of integration:

$$\int \cos^3(x)dx.$$

(c) Find the antiderivative with constant of integration:

$$\int \frac{dx}{x^2 + x}.$$

(d) Find the number:

$$\int_{x=0}^{1} x(x^2+1)^7 dx.$$

(e) Find the number:

$$\int_{x=0}^{\frac{\pi}{2}} (2x+1)\sin(x)dx.$$

(f) Find the number:

$$\int_{x=0}^{2} \left(1+x^2\right)^{-\frac{3}{2}}dx.$$

2. (50 points)
Consider the first quadrant region bounded by the curve

$$y = e^x$$

and the line

$$y = 1 + (e - 1)x.$$

This region runs from $x = 0$ to $x = 1$.
Consider also the solid obtained by revolving the given region about the y-axis. (It looks like a bowl).

(a) Write a definite integral(s) for the length of the boundary of the given region. (This is the sum of the lengths of the curve and the line). You do not need to evaluate the integral(s).

(b) Write a definite integral(s) for the surface area of the given solid. (This is the sum of the areas of the surfaces obtained by revolving the curve and by revolving the line). You do not need to evaluate the integral(s).

(c) Write a definite integral for the volume of the given solid. You do not need to evaluate the integral.

3. (10 points: Extra Credit... you may skip this problem)
The functions f and g satisfy:

   (a) $f''(x) = 3f(x)$ and $g(x) = 2g''(x)$,
   (b) $f(5) = 3$ and $g(5) = \frac{1}{2}$,
   (c) $f'(5) = g'(5) = 1$ and
   (d) $f(0) = g(0) = f'(0) = g'(0) = 0$.

Find:

$$\int_{x=0}^{5} f(x)g(x)dx.$$