Math 21B Practice Midterm II Spring 2025

You may use one page of notes but not a calculator or textbook. Please do not simplify your answers.

1. (50 points: Integration)

   (a) Find the number:
   $$\int_{x=0}^{1} \frac{dx}{\sqrt{4-x^2}}.$$

   (b) Find the number:
   $$\int_{x=-1}^{1} \frac{xdx}{4-x^2}.$$

   (c) Find the number:
   $$\int_{x=-1}^{1} \frac{dx}{4-x^2}.$$

   (d) Find the antiderivative with constant of integration:
   $$\int x\ln(x)dx.$$

   (e) Find the antiderivative with constant of integration:
   $$\int \frac{\sin(x)}{1-\sin^2(x)}dx.$$

   (f) Find the antiderivative with constant of integration:
   $$\int \frac{e^{-\sqrt{x}}dx}{\sqrt{x}}.$$

2. (50 points)
Consider the curve $y = \sin(x)$ between $x = 0$ and $x = \frac{\pi}{2}$ (so the ends are at $(0, 0)$ and $(\frac{\pi}{2}, 1)$).

   (a) Write a definite integral for the length of the given curve.

   (b) Write a definite integral for the area of the surface obtained by rotating the given curve about the axis $x = -2$.

   (c) Consider the the region bounded above by the given curve and below by the line segment connecting its two ends. Write a definite integral for the volume of the object obtained by rotating this region about the y-axis.

3. (10 points: Extra Credit... you may skip this problem)
(Dough to sprinkles ratio)
Find the ratio of the volume to the surface area of a volume of revolution obtained by rotating a circle of radius r about a line with distance R from the center of the circle.

Basic and Trigonometric Integrals

$\int x^n dx$	$=$	$\frac{1}{n+1}x^{n+1} + C$
$\int x^{-1} dx$	$=$	$\ln \lvert x \rvert + C$
$\int e^x dx$	$=$	$e^x + C$
$\int \sin(x) dx$	$=$	$-\cos(x) + C$
$\int \cos(x) dx$	$=$	$\sin(x) + C$
$\int \frac{dx}{\sqrt{1-x^2}} dx$	$=$	$\arcsin(x) + C = -\arccos(x) + C$
$\int \sec^2(x) dx$	$=$	$\tan(x) + C$
$\int \tan(x) dx$	$=$	$\ln \lvert \sec(x) \rvert + C$
$\int \csc^2(x) dx$	$=$	$-\cot(x) + C$
$\int \cot(x) dx$	$=$	$\ln \lvert \sin(x) \rvert + C$
$\int \frac{dx}{1+x^2} dx$	$=$	$\arctan(x) + C = -\text{arccot}(x) + C$
$\int \sec(x)\tan(x) dx$	$=$	$\sec(x) + C$
$\int \sec(x) dx$	$=$	$\ln \lvert \sec(x) + \tan(x) \rvert + C$
$\int \csc(x)\cot(x) dx$	$=$	$-\csc(x) + C$
$\int \csc(x) dx$	$=$	$-\ln \lvert \csc(x) + \cot(x) \rvert + C$
$\int \frac{dx}{\lvert x \rvert \sqrt{x^2-1}} dx$	$=$	$\text{arcsec}(x) + C = -\text{arccsc}(x) + C$

Trigonometric Identities

$\cos^2(x) + \sin^2(x)$	$=$	1
$\sec^2(x) - \tan^2(x)$	$=$	1
$\cos^2(x)$	$=$	$\frac{1}{2}[1 + \cos(2x) + 1]$
$\sin^2(x)$	$=$	$\frac{1}{2}[1 - \cos(2x) + 1]$
$\cos(a+b)$	$=$	$\cos(a)\cos(b) - \sin(a)\sin(b)$
$\sin(a+b)$	$=$	$\sin(a)\cos(b) + \cos(a)\sin(b)$