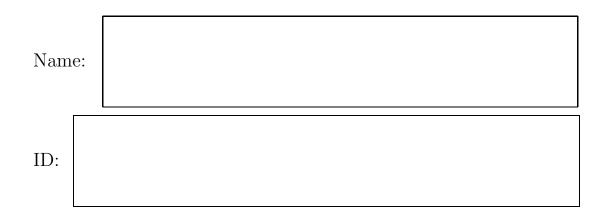
Math 21B Midterm III Spring 2025 Wed May 28 3:10-4:00

You may use one page of notes but not a calculator or textbook. Please do not simplify your answers.



1. (10 points)

It takes a force of 10N to extend a spring 5m. How much work does this extension require?

Assume the spring follows Hooke's law with force F = kx for some constant k and extension length x.

2. (20 points)

Consider an initial value problem $\frac{dy}{dt} = \sqrt{yt}$ with y(0) = 4. Find y(9).

- 3. Consider the region bounded above by the curve $y = 1 x^4$ and below by the x-axis between x = -1 and x = 1.
 - (a) (15 points)Find the area of the given region.

(b) (15 points)Find the center of mass of the given region.

4. (a) (10 points) Find the improper integral

$$\int_0^1 \frac{dx}{x^2}$$

or determine that it diverges.

(b) (10 points)

Use a comparison test to determine if the improper integral

$$\int_0^1 \frac{dx}{x^2 \cos(x)}$$

converges or diverges.

Hint: Use (a) for (b).

5. (20 points)

Find an equation for the line tangent at the point where t = 1 to the curve given parametrically by $x(t) = t + t^3$ and $y(t) = t \ln(t)$.

6. (Extra Credit)(10 points):

Consider the infinite region bounded above by $y = x^{-k}$, below by the x-axis and between x = 1 and $x = \infty$. Consider also the infinite solid obtained by revolving this region about the x-axis.

(a) For which values of the constant k is the area of the region finite?

(b) For which values of the constant k is the volume of the solid finite?

Asside: Does the region have a centroid for the values of k where the area of the region is infinite but the volume of the solid is finite?