Math 21B Midterm III Spring 2025 Wed May 28 3:10-4:00

You may use one page of notes but not a calculator or textbook.
Please do not simplify your answers.

Name:

ID:

1. (10 points)
   It takes a force of 10N to extend a spring 5m.
   How much work does this extension require?

   Assume the spring follows Hooke's law with force $F = kx$ for some constant k and extension length x.

2. (20 points)
   Consider an initial value problem $\frac{dy}{dt} = \sqrt{yt}$ with $y(0) = 4$.
   Find $y(9)$.

3. Consider the region bounded above by the curve $y = 1 - x^4$ and below by the x-axis between $x = -1$ and $x = 1$.

   (a) (15 points)
   Find the area of the given region.

   (b) (15 points)
   Find the center of mass of the given region.

4. (a) (10 points)
Find the improper integral

$$\int_0^1 \frac{dx}{x^2}$$

or determine that it diverges.

(b) (10 points)
Use a comparison test to determine if the improper integral

$$\int_0^1 \frac{dx}{x^2 \cos(x)}$$

converges or diverges.

Hint: Use (a) for (b).

5. (20 points)
Find an equation for the line tangent at the point where $t = 1$ to the curve given parametrically by $x(t) = t + t^3$ and $y(t) = t \ln(t)$.

6. (Extra Credit)(10 points):
Consider the infinite region bounded above by $y = x^{-k}$, below by the x-axis and between $x = 1$ and $x = \infty$. Consider also the infinite solid obtained by revolving this region about the x-axis.

   (a) For which values of the constant k is the area of the region finite?

   (b) For which values of the constant k is the volume of the solid finite?

   Asside: Does the region have a centroid for the values of k where the area of the region is infinite but the volume of the solid is finite?