## Math 21B Practice Midterm III Spring 2025

You may use one page of notes but not a calculator or textbook. Please do not simplify your answers.

- 1. Consider a 4m deep wedge shaped pool with rectangular top 5m by 12m and triangular side.
  - (a) Find the work required to empty the initially full pool by pumping all the water out the top. The weight density of the water is  $\omega = 50 \frac{N}{m^3}$ .
  - (b) There is an underground maintainance room at the deep end with a rectangular door that is 2m tall and 1m wide. (So when the pool is full the top of the door is under 2m of water). Find the force on the door when the pool is full.
- 2. Consider the initial value problem:  $\frac{dy}{dt} = t^2 y$  with y(0) = 5. Find y(3).
- 3. Consider the region bounded above by the curve  $y = 1 x^2$  and below by the x-axis between x = -1 and x = 1.
  - (a) Find the area of the given region.
  - (b) Find the center of mass of the given region.
  - (c) Find the volumes of the solids of revolution obtained by revolving the given region about each of the following:
    - i. the *x*-axis,
    - ii. the line x = 1,
    - iii. the line y = 1.

Hint: Use (a) and (b) for (c).

4. (a) Find the improper integral

$$\int_0^\infty e^{-x} dx.$$

(b) Determine whether the improper integral

$$\int_0^\infty (1+\sin(x))e^{-x}dx$$

converges or diverges.

Hint: Use (a) for (b).

5. Find  $\frac{dy}{dx}$  at t = 1 if  $x(t) = te^t$  and  $y(t) = t + \sin(\pi t)$ .