Section 5.5

3.) $\int 2x(x^2+5)^{-4}\,dx$ (Let $u = x^2+5 \rightarrow du = 2x\,dx$)

$= \int u^{-4}\,du = \frac{-1}{3}u^{-3} + c = \frac{-1}{3}(x^2+5)^{-3} + c$

5.) $\int (3x+2)(3x^2+4x)^4\,dx$ (Let $u = 3x^2+4x \rightarrow$
$du = (6x+4)\,dx = 2(3x+2)\,dx \rightarrow \frac{1}{2}du = (3x+2)\,dx$)

$= \frac{1}{2}\int u^4\,du = \frac{1}{2}\cdot\frac{1}{5}u^5 + c = \frac{1}{10}(3x^2+4x)^5 + c$

6.) $\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx$ (Let $u = 1+\sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\,dx$
$\rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$)

$= 2\int u^{1/3}\,du = 2\cdot\frac{3}{4}u^{4/3} + c = \frac{3}{2}(1+\sqrt{x})^{4/3} + c$

7.) $\int \sin 3x\,dx$ (Let $u = 3x \rightarrow du = 3\,dx$
$\rightarrow \frac{1}{3}du = dx$)

$= \frac{1}{3}\int \sin u\,du = \frac{-1}{3}\cos u + c = \frac{-1}{3}\cos 3x + c$

10.) $\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\,dt$ (Let $u = 1-\cos\frac{t}{2} \rightarrow$
$du = -\left(-\sin\frac{t}{2}\cdot\frac{1}{2}\right)dt = \frac{1}{2}\sin\frac{t}{2}\,dt \rightarrow$
$2\,du = \sin\frac{t}{2}\,dt$)

$= 2\int u^2\,du = 2\cdot\frac{u^3}{3} + c = \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + c$

11.) $\int \frac{9r^2}{\sqrt{1-r^3}}\,dr$ (Let $u = 1-r^3 \rightarrow du = -3r^2\,dr$
$\rightarrow \frac{-1}{3}du = r^2\,dr$)

$= 9\cdot\left(\frac{-1}{3}\right)\int \frac{1}{\sqrt{u}}\,du = -3\int u^{-1/2}\,du$

$= -3\cdot\frac{u^{1/2}}{1/2} + c = -6\sqrt{1-r^3} + c$

12.) $\int 12(y^4+4y^2+1)^2(y^3+2y)\,dy$

(Let $u=y^4+4y^2+1 \rightarrow du=(4y^3+8y)\,dy \rightarrow$
$du=4(y^3+2y)\,dy \rightarrow \frac{1}{4}du=(y^3+2y)\,dy$)

$= 12\left(\frac{1}{4}\right)\int u^2\,du = 3\cdot\frac{u^3}{3}+c = (y^4+4y^2+1)^3+c$

14.) $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$ (Let $u=\frac{1}{x} \rightarrow$
$du=\frac{-1}{x^2}dx \rightarrow -du=\frac{1}{x^2}dx$)

$= -\int \cos^2 u\,du$ (Recall: $\cos 2\theta = 2\cos^2\theta - 1$
$\rightarrow \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$)

$= -\int \frac{1}{2}(1+\cos 2u)\,du$

$= -\frac{1}{2}\left(u+\frac{1}{2}\sin 2u\right)+c$

$= -\frac{1}{2}\left(\frac{1}{x}+\frac{1}{2}\sin\frac{2}{x}\right)+c$

15.) $\int \csc^2 2\theta\cdot\cot 2\theta\,d\theta$ (Let $u=\cot 2\theta \rightarrow$
$du=-\csc^2 2\theta\cdot 2\,d\theta \rightarrow \frac{-1}{2}du=\csc^2 2\theta\,d\theta$)

$= \frac{-1}{2}\int u\,du = \frac{-1}{2}\cdot\frac{u^2}{2}+c = -\frac{1}{4}\cot^2 2\theta + c$

17.) $\int \sqrt{3-2s}\,ds$ (Let $u=3-2s \rightarrow$
$du=-2\,ds \rightarrow \frac{-1}{2}du=ds$)

$= \frac{-1}{2}\int\sqrt{u}\,du = \frac{-1}{2}\cdot\frac{u^{3/2}}{3/2}+c = -\frac{1}{3}(3-2s)^{3/2}+c$

22.) $\int\sqrt{\sin x}\cdot\cos^3 x\,dx = \int\sqrt{\sin x}\cdot\cos^2 x\cdot\cos x\,dx$

$= \int\sqrt{\sin x}\,(1-\sin^2 x)\cdot\cos x\,dx$ (Let $u=\sin x \rightarrow$

$= \int\sqrt{u}\,(1-u^2)\,du$ $du=\cos x\,dx$)

$= \int(u^{1/2}-u^{5/2})\,du = \frac{2}{3}u^{3/2}-\frac{2}{7}u^{7/2}+c$

$= \frac{2}{3}(\sin x)^{3/2}-\frac{2}{7}(\sin x)^{7/2}+c$

23.) $\int \sec^2(3x+2)\,dx$ (Let $u = 3x+2 \rightarrow$
$du = 3\,dx \rightarrow \frac{1}{3}du = dx$)
$= \frac{1}{3}\int \sec^2 u\,du = \frac{1}{3}\tan u + c = \frac{1}{3}\tan(3x+2)+c$

24.) $\int \tan^2 x \sec^2 x\,dx$ (Let $u = \tan x \rightarrow$
$du = \sec^2 x\,dx$)
$= \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}(\tan x)^3 + c$

25.) $\int \sin^5 \frac{x}{3} \cos\frac{x}{3}\,dx$ (Let $u = \sin\frac{x}{3} \rightarrow$
$du = \cos\frac{x}{3}\cdot\frac{1}{3}dx \rightarrow 3\,du = \cos\frac{x}{3}\,dx$)
$= 3\int u^5 du = 3\cdot\frac{1}{6}u^6 + c = \frac{1}{2}\left(\sin\frac{x}{3}\right)^6 + c$

28.) $\int r^4\left(7-\frac{1}{10}r^5\right)^3 dr$ (Let $u = 7-\frac{1}{10}r^5 \rightarrow$
$du = -\frac{1}{2}r^4\,dr \rightarrow -2\,du = r^4\,dr$)
$= -2\int u^3 du = -2\cdot\frac{1}{4}u^4 + c = \frac{-1}{2}\left(7-\frac{1}{10}r^5\right)^4 + c$

29.) $\int x^{1/2}\sin(x^{3/2}+1)\,dx$ (Let $u = x^{3/2}+1 \rightarrow$
$du = \frac{3}{2}x^{1/2}dx \rightarrow \frac{2}{3}du = x^{1/2}dx$)
$= \frac{2}{3}\int \sin u\,du = \frac{2}{3}\cdot -\cos u + c = -\frac{2}{3}\cos(x^{3/2}+1)+c$

31.) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$ (Let $u = \cos(2t+1) \rightarrow$
$du = -\sin(2t+1)\cdot 2\,dt \rightarrow \frac{-1}{2}du = \sin(2t+1)\,dt$)
$= \frac{-1}{2}\int \frac{1}{u^2}du = \frac{-1}{2}\cdot\frac{u^{-1}}{-1} + c = \frac{1}{2}(\cos(2t+1))^{-1} + c$

32.) $\int \frac{\sec z \tan z}{\sqrt{\sec z}}\,dz$ (Let $u = \sec z \rightarrow$
$du = \sec z \tan z\,dz$)

$$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2(\sec z)^{1/2} + C$$

33.) $\int \frac{1}{t^2}\cos\left(\frac{1}{t}-1\right)dt$ (Let $u = \frac{1}{t}-1 \rightarrow$

$du = -\frac{1}{t^2}\,dt \rightarrow -du = \frac{1}{t^2}\,dt$)

$$= -\int \cos u\,du = -\sin u + C = -\sin\left(\frac{1}{t}-1\right) + C$$

36.) $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\cdot\sin^2\sqrt{\theta}}\,d\theta$ (Let $u = \sin\sqrt{\theta} \rightarrow$

$du = \cos\sqrt{\theta}\cdot\frac{1}{2\sqrt{\theta}}\,d\theta \rightarrow 2\,du = \frac{\cos\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$)

$$= 2\int \frac{1}{u^2}\,du = 2\int u^{-2}\,du = -2u^{-1} + C$$

$$= -2(\sin\sqrt{\theta})^{-1} + C$$

38.) $\int \sqrt{\frac{x-1}{x^5}}\,dx = \int \sqrt{\frac{x-1}{x^4\cdot x}}\,dx$ (assume $x \geq 1$.)

$$= \int \frac{1}{x^2}\sqrt{\frac{x-1}{x}}\,dx = \int \frac{1}{x^2}\cdot\sqrt{1-\frac{1}{x}}\,dx$$

(Let $u = 1-\frac{1}{x} \rightarrow du = \frac{1}{x^2}\,dx$)

$$= \int \sqrt{u}\,du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3}\left(1-\frac{1}{x}\right)^{3/2} + C$$

40.) $\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$ (Let $u = 1-x^{-2} \rightarrow$

$du = 2x^{-3}\,dx \rightarrow \frac{1}{2}\,du = \frac{1}{x^3}\,dx$)

$$= \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} + C$$

41.) $\int \sqrt{\frac{x^3-3}{x^{11}}}\,dx = \int \sqrt{\frac{x^3-3}{x^8x^3}}\,dx = \int \frac{1}{x^4}\sqrt{\frac{x^3}{x^3}-\frac{3}{x^3}}\,dx$

$= \int \frac{1}{x^4}\sqrt{1-3x^{-3}}\,dx$ (Let $u = 1-3x^{-3} \rightarrow$

$du = 9x^{-4}dx \rightarrow \frac{1}{9}du = \frac{1}{x^4}dx$)

$= \frac{1}{9}\int \sqrt{u}\,du = \frac{1}{9}\cdot\frac{2}{3}u^{3/2}+c = \frac{2}{27}(1-3x^{-3})^{3/2}+c$

42.) $\int \sqrt{\frac{x^4}{x^3-1}}\,dx = \int \frac{\sqrt{x^4}}{\sqrt{x^3-1}}\,dx = \int \frac{x^2}{\sqrt{x^3-1}}\,dx$

(Let $u = x^3-1 \rightarrow du = 3x^2dx \rightarrow \frac{1}{3}du = x^2dx$)

$= \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\int u^{-1/2}du = \frac{1}{3}\cdot 2u^{1/2}+c = \frac{2}{3}(x^3-1)^{1/2}+c$

43.) $\int x(x-1)^{10}dx$ (Let $u = x-1 \rightarrow du = dx$ and $x = u+1$)

$= \int (u+1)u^{10}du = \int (u^{11}+u^{10})\,du = \frac{1}{12}u^{12}+\frac{1}{11}u^{11}+c$

$= \frac{1}{12}(x-1)^{12}+\frac{1}{11}(x-1)^{11}+c$

44.) $\int x\sqrt{4-x}\,dx$ (Let $u = 4-x \rightarrow du = -dx$

$\rightarrow -du = dx$ and $x = 4-u$)

$= -\int (4-u)u^{1/2}du = -\int (4u^{1/2}-u^{3/2})\,du$

$= -\left(4\cdot\frac{2}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right)+c$

$= -\frac{8}{3}(4-x)^{3/2}+\frac{2}{5}(4-x)^{5/2}+c$

46.) $\int (x+5)(x-5)^{1/3}dx$ (Let $u = x-5 \rightarrow du = dx$

and $x = u+5$)

$$= \int ((u+5)+5)u^{1/3}\,du = \int (u+10)u^{1/3}\,du$$

$$= \int (u^{4/3}+10u^{1/3})\,du = \frac{3}{7}u^{7/3} + 10\cdot\frac{3}{4}u^{4/3} + c$$

$$= \frac{3}{7}(x-5)^{7/3} + \frac{15}{2}(x-5)^{4/3} + c$$

47.) $\int x^3\sqrt{x^2+1}\,dx$ (Let $u = x^2+1 \rightarrow$
$du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$
and $x^2 = u-1$)

$$= \int x^2\cdot x\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\cdot x\,dx$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int (u^{3/2} - u^{1/2})\,du$$

$$= \frac{1}{2}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + c = \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + c$$

51.) $\int \cos x \cdot e^{\sin x}\,dx$ (Let $u = \sin x \rightarrow$
$du = \cos x\,dx$)

$$= \int e^u\,du = e^u + c = e^{\sin x} + c$$

54.) $\int \frac{1}{x^2}e^{1/x}\sec(1+e^{1/x})\cdot\tan(1+e^{1/x})\,dx$
(Let $u = 1+e^{1/x} \rightarrow du = e^{1/x}\cdot\frac{-1}{x^2}\,dx \rightarrow$
$-du = \frac{1}{x^2}e^{1/x}\,dx$)

$$= -\int \sec u\tan u\,du = -\sec u + c$$

$$= -\sec(1+e^{1/x}) + c$$

55.) $\int \frac{dx}{x\ln x}$ (Let $u = \ln x \rightarrow du = \frac{1}{x}dx$)

$$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|\ln x| + c$$

58.) $\int \frac{1}{x\sqrt{x^4-1}}\,dx = \int \frac{x}{x^2\cdot\sqrt{(x^2)^2-1}}\,dx$

(Let $u = x^2 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \frac{1}{u\sqrt{u^2-1}}\,du = \frac{1}{2}\operatorname{arcsec} u + c$

$= \frac{1}{2}\operatorname{arcsec} x^2 + c$

59.) $\int \frac{5}{9+4r^2}\,dr = 5\int \frac{1}{9(1+\frac{4}{9}r^2)}\,dr$

$= \frac{5}{9}\int \frac{1}{1+(\frac{2}{3}r)^2}\,dr$ (Let $u = \frac{2}{3}r \rightarrow du = \frac{2}{3}dr$

$\rightarrow \frac{3}{2}du = dr$)

$= \frac{5}{9}\cdot\frac{3}{2}\int \frac{1}{1+u^2}\,du = \frac{5}{6}\arctan u + c$

$= \frac{5}{6}\arctan\left(\frac{2}{3}r\right) + c$

65.) $\int \frac{1}{(\arctan y)\cdot(1+y^2)}\,dy$ (Let $u = \arctan y \rightarrow$

$du = \frac{1}{1+y^2}\,dy$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|\arctan y| + c$

69.) $\int \frac{(2r-1)\cos\sqrt{3(2r-1)^2+6}}{\sqrt{3(2r-1)^2+6}}\,dr$

(Let $u = \sqrt{3(2r-1)^2+6} \rightarrow$

$du = \frac{1}{\cancel{2}}(3(2r-1)^2+6)^{-1/2}\cdot 6(2r-1)\cdot\cancel{2}\,dr \rightarrow$

$\frac{1}{6}du = \frac{(2r-1)}{\sqrt{3(2r-1)^2+6}}\,dr$)

$= \frac{1}{6}\int \cos u \, du = \frac{1}{6}\sin u + c$

$= \frac{1}{6}\sin\sqrt{3(2r-1)^2+6} + c$

70.) $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cdot\cos^3\sqrt{\theta}}}\,d\theta = \int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cdot\cos^{3/2}\sqrt{\theta}}\,d\theta$

(Let $u = \cos\sqrt{\theta} \rightarrow du = -\sin\sqrt{\theta}\cdot\frac{1}{2\sqrt{\theta}}\,d\theta$

$\rightarrow -2\,du = \frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$)

$= -2\int \frac{1}{u^{3/2}}\,du = -2\int u^{-3/2}\,du = -2\cdot -2u^{-1/2} + c$

$= 4\,(\cos\sqrt{\theta})^{-1/2} + c$

71.) $\frac{ds}{dt} = 12t(3t^2-1)^3 \rightarrow s = \int 12t(3t^2-1)^3\,dt$

(Let $u = 3t^2-1 \rightarrow du = 6t\,dt \rightarrow 2du = 12t\,dt$)

$= 2\int u^3\,du = 2\cdot\frac{1}{4}u^4 + c = \frac{1}{2}(3t^2-1)^4 + c \rightarrow$

$s = \frac{1}{2}(3t^2-1)^4 + c$ and $t=1,\ s=3 \rightarrow$

$3 = \frac{1}{2}(2)^4 + c = 8 + c \rightarrow c = -5$ so that

$s = \frac{1}{2}(3t^2-1)^4 - 5$

78.) $y'' = 4\sec^2 2x\tan 2x \rightarrow$

$y' = \int 4\sec^2 2x\tan 2x\,dx$ (Let $u = \tan 2x \rightarrow$

$du = \sec^2 2x\cdot 2\,dx \rightarrow 2\,du = 4\sec^2 2x\,dx$)

$= 2\int u\,du = u^2 + c = \tan^2 2x + c \rightarrow$

$Y' = \tan^2 2X + C$ and $X=0, Y'=4 \rightarrow$
$4 = \cancel{\tan^2} 0 + C \rightarrow C = 4$ so that
$\underline{Y' = \tan^2 2X + 4} \rightarrow Y = \int (\tan^2 2X + 4)\, dx$
$= \int ((\sec^2 2X - 1) + 4)\, dx = \int (\sec^2 2X + 3)\, dx$
$= \frac{1}{2} \tan 2X + 3X + C \rightarrow Y = \frac{1}{2} \tan 2X + 3X + C$
and $X = 0, Y = -1 \rightarrow -1 = \frac{1}{2} \cancel{\tan} 0 + 3(0) + C$
$\rightarrow C = -1$ and $\underline{Y = \frac{1}{2} \tan 2X + 3X - 1}$.

80.) $S'' = \pi^2 \cos \pi t$ m./sec.^{2} $\rightarrow$
$S' = \pi^2 \cdot \frac{1}{\pi} \sin \pi t + C = \pi \sin \pi t + C$ and
$t = 0, S' = 8$ m./sec. $\rightarrow 8 = \pi \cancel{\sin} 0 + C \rightarrow$
$C = 8 \rightarrow \underline{S' = \pi \sin \pi t + 8} \rightarrow$

$S = \cancel{\pi} \cdot \frac{-1}{\cancel{\pi}} \cos \pi t + 8t + C$ and $t = 0, S = 0$ m. $\rightarrow$
$0 = - \cancel{\cos} 0 + 8(0) + C \rightarrow C = 1 \rightarrow$
$\underline{S = -\cos \pi t + 8t + 1}$; if $t = 1$ sec. $\rightarrow$
$S = - \cancel{\cos} \pi + 8(1) + 1 = 10$ m.