1. Infinite Groups F18.1

Fall 2018 Problem 1. Show that a free group of rank ≥ 2 is not nilpotent.

1.1. Ideas F18.1.

- Recall the difference between nilpotent and solvable.
- Note that if F_2 is not nilpotent then neither is $F_{n>2}$.
- Try explicitly writing elements in successive commutators (lower central series).
- This exercise seems inductive so try embedding F_2 into $[F_2, F_2]$.
- Quotients of nilpotent groups should again be nilpotent by the correspondence theorem so any group with two generators which is not nilpotent would suffice.
- Symmetric groups are two generated.

2. Infinite Groups F16.1

Fall 2016 Problem 1. Let F_n be the free group on n generators with $n \geq 2$. Prove that the center $Z(F)$ of F is trivial.

2.1. Ideas F16.1.

- It suffices to work with F_2.
- Express any element as a word and explicitly find another with which it does not commute.

3. Infinite Groups F15.5

Fall 2015 Problem 5. Show that two free groups are isomorphic if and only if they have equal ranks.

3.1. Ideas F15.5.

- If two are isomorphic this induces an isomorphism between their Abelianizations.
- This further induces an isomorphism between these Abelianizations tensored over $\mathbb{Z}$ with C_2.
- These groups have cardinality 2^r with r the rank of the free group.