1. Linear Algebra F15.3

Fall 2015 Problem 3: What is the smallest possible n for which there is an n by n real matrix M which has both:

(1) the rank of M^2 is smaller than the rank of M,
(2) M leaves infinitely many length one vectors fixed.

1.1. Ideas F15.3.

- Fixed vectors are eigenvectors with eigenvalue one. Two independent ones give infinitely many of length one.
- The rank of M^2 is related to the number and size of Jordan blocks with eigenvalue 0.

2. Linear Alg S16.5

Spring 2016 Problem 5 Find all possible Jordan canonical forms for a matrix $A = T((123))$ if T is a two dimensional complex representation of the symmetric group S_3.

2.1. Ideas S16.5.

- Since $(123)^3 = I$ one has $A^3 = I$.
- This makes A diagonalizable with three possible eigenvalues.

3. Linear Alg S15.1

Spring 2015 Problem 1: Show that if M is a nondiagonalizable complex matrix and M^n is diagonalizable then $\det(M)$=0.

3.1. Ideas S15.1.

- Work out the Jordan form of a power of a Jordan block.

4. Linear Alg F17.3

Fall 2017 Problem 3: Let $M_n(\mathbb{R})$ denote the ring of $n \times n$ matrices over $\mathbb{R}$, and consider a (possibly non-unital) ring homomorphism $f : M_{n+1}(\mathbb{R}) \to M_n(\mathbb{R})$. Can f be nonzero?

- Consider determinants.