1. Tensors F18.2

Fall 2018 Problem 2. Let $\mathbb{H}$ be the real quarternions. Then $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{H}$ is isomorphic to which of the following rings? Prove your answer.

(a) $\mathbb{C} \times \mathbb{C}$
(b) $\mathbb{C} \times \mathbb{C} \times \mathbb{C} \times \mathbb{C}$
(c) $\mathbb{M}_2(\mathbb{C})$
(d) $\mathbb{M}_2(\mathbb{R})$
(e) $\mathbb{M}_2(\mathbb{H})$
(f) $\mathbb{M}_2(\mathbb{R}) \times \mathbb{M}_2(\mathbb{R})$

1.1. Ideas F18.2.

- This comes from Brauer groups.
- Consider the $\mathbb{R}$- dimensions.
- Consider the centers.

2. Tensors S13.2

Spring 2013 Problem 2. Consider an attempt to make an $\mathbb{R}$-linear map

$$f : \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C} \to \mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$$

or

$$\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \to \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C},$$

in either direction given be the formula $f(x \otimes y) = x \otimes y$. In which direction is this map well defiined? Is it then surjective? Is it injective?

2.1. Ideas F16.6.

- If $f : R \to S$ is a ring map and M and N are S modules then via f they are also R modules and f induces a map from $M \otimes_R N$ to $M \otimes_S N$.