Fall 2016 Problem 2: Let G be a finite group that acts transitively on a set X of cardinality ≥ 2. Show that there exists an element $g \in G$ which acts on X without any fixed points. Is the same true if G is infinite?

Spring 2016 Problem 4: Prove that every finite group G of order > 2 has a nontrivial automorphism.

Fall 2017 Problem 6: Prove that if p is a prime number, then every group G with p^2 elements is abelian.

Fall 2001 Problem 2:

(1) Give an example of a group and a subroup which is not normal.
(2) Show that every group of order 33 has a normal subgroup of order 11.

Winter 2002 Problem 9: Assume that H and K are normal subgroups of G with trivial intersection.

(1) Show that if $h \in H$ and $k \in K$ then $hk = kh$.
(2) Show that if $h_i \in H$ and $k_i \in K$ with $h_1k_1 = h_2k_2$ then $h_1 = h_2$ and $k_1 = k_2$.
(3) Show that $HK = \{hk | h \in H, k \in K\}$ is a group isomorphic to $H \times K$.

Fall 2014 Problem 2: Show that every subgroup of index equal to the smallest prime dividing the order of a group in normal.

Fall 2002 Problem 2 Consider a finite group G with subgroup S and normalizer $N = N_G(S)$.

(1) Show that N is a subgroup of G.
(2) Show that if $S \leq K \leq G$ are groups then S is normal in K iff $K \leq N$.
(3) Show that the number of subgroups of G conjugate to S is the index $[G : N]$.
(4) Find $N_{S_4}(\langle(1, 2, 3, 4)\rangle)$.