Fall 2016 Problem 6. Show that the $\mathbb{R}$-modules $L = \mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ and $L = \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$ are not isomorphic.

Fall 2013 Problem 2. Consider an attempt to make an $\mathbb{R}$-linear map

$$f : \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C} \to \mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$$

or

$$\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \to \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C},$$

in either direction given be the formula $f(x \otimes y) = x \otimes y$. In which direction is this map well defiined? Is it then surjective? Is it injective?

Fall 2018 Problem 2. Let $\mathbb{H}$ be the real quarternions. Then $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{H}$ is isomorphic to which of the following rings? Prove your answer.

(a) $\mathbb{C} \times \mathbb{C}$
(b) $\mathbb{C} \times \mathbb{C} \times \mathbb{C} \times \mathbb{C}$
(c) $\mathcal{M}_2(\mathbb{C})$
(d) $\mathcal{M}_2(\mathbb{R})$
(e) $\mathcal{M}_2(\mathbb{H})$
(f) $\mathcal{M}_2(\mathbb{R}) \times \mathcal{M}_2(\mathbb{R})$

Fall 2014 Problem 6. If R is a commutative ring with identity, and S a multiplicative subset, then every ideal J of $S^{-1}R$ is of the form $S^{-1}I$ for some ideal I of R. Is I uniquely determined by J? Why or why not?

Winter 2008 Problem 5: What are the possible orders of the tensor product as $\mathbb{Z}$ modules of two Abelian groups which each have order 25?

Winter 2009 Problem 4: Let P be any set of primes in $\mathbb{Z}$. Find a commutative ring R containing $\mathbb{Z}$ for which the irreducible elements are precisely units times elements of P.
(Hint: consider $\mathbb{Z}[\frac{1}{2}] = \{\frac{a}{2^r}\} \subseteq \mathbb{Q}$).

Fall 2009 Problem 5: Show that $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ and $\mathbb{C} \oplus \mathbb{C}$ are isomorphic as $\mathbb{C}$-algebras.

Spring 2014 Problem 6: Consider the field $\mathbb{C}(x)$ of complex rational functions as a $\mathbb{C}$-algebra and the tensor product of two copies $\mathbb{C}(x) \otimes_{\mathbb{C}} \mathbb{C}(y)$ as another $\mathbb{C}$-algebra. Is this also a field?

Fall 2018 Problem 4. Solve the following:

(a) Prove that $R/I \otimes_R R/J \cong R/(I+J)$ for R a commutative ring, and $I, J \subset R$ are ideals.
(b) Find the dimension of $\mathbb{Q}[x,y]/(x^2+y^2) \otimes_{\mathbb{Q}[x,y]} \mathbb{Q}[x,y]/(x+y^3)$ as a vector space over $\mathbb{Q}$, or explain why it is infinite.