Fall 2017 Problem 1 Suppose that $F \subseteq K$ is an inclusion of fields and let $\alpha, \beta \in K$ be two elements which are algebraic over F. Show that $\alpha + \beta$ is also algebraic over F.

Fall 2013 Problem 6. Compute $[\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}]$ and find a basis for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.

Fall 2016 Problem 5 Let F be a finite field and let L be a subfield of F generated by elements of the form x^3 for all $x \in F$. Prove that if $L \neq F$, the F has exactly 4 elements.

Fall 2013 Problem 5 Is it possible to have a field extension $F \subseteq K$ with $[K : F] = 2$, where both fields F and K are isomorphic to the field $\mathbb{Q}(x)$?

Fall 2017 Problem 2. Let $f \in \mathbb{Q}[x]$ be the minimal polynomial of $1 + \sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$, and let K be the splitting field for f over $\mathbb{Q}$. What is $[K : \mathbb{Q}]$ and what is $\mathrm{Gal}(K/\mathbb{Q})$? (Note that you are not required to find f.)

Fall 2015 Problem 6. Find the $\mathbb{Q}$ dimension of the splitting field over $\mathbb{Q}$ of $x^5 - 3$.

Spring 2010 Problem 6: Find an irreducible polynomial over $\mathbb{Q}$ for which the order of the Galois group is not the factorial of the degree of the polynomial.

Fall 2009 Problem 2: The field extensions $\mathbb{Q}(\sqrt{\sqrt{2}})/\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{2})/\mathbb{Q}$ are both Galois. Show that $\mathbb{Q}(\sqrt{\sqrt{2}})/\mathbb{Q}$ is not Galois.

Fall 2004 Problem 6: Construct infinitely many nonisomorphic quadratic extensions of $\mathbb{Q}$ and use these to show that the Abelianization of $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$ is not finitely generated.