**Spring 2014 Problem 2:** Consider the four dimensional real vector space

$$V = \{f : \mathbb{Z}/4\mathbb{Z} \to \mathbb{R}\}$$

with the $\mathbb{R}[x]$-module structure given by shifting so that $(xf)(r) = f(r+1)$. Find a direct sum decomposition of $V$ into irreducible $\mathbb{R}[x]$ modules.

**Spring 2014 Problem 4:** Give an example of a ring $R$ and an $R$ module which is projectve but not free.

**Spring 2015 Problem 4:** Find a quotient ring of $\mathbb{Z}[x]$ which is a pid but not a field.

**Spring 2015 Problem 5:** Let $R = \mathbb{Q}[X]/(X^3 - 2)$.

(1) Is $R$ a field? Explain.

(2) Run the extended Euclidean algorithm on $X^3 - 2$ and $X^2 - X + 1$ to find polynomials $A(x)$ and $B(x)$ with

$$A(X)(X^3 - 2) + B(X)(X^2 - X + 1) = \gcd(X^3 - 2, X^2 - X + 1).$$

(3) Does $[X^2 - X + 1]$ have a multiplicative inverse in $R$? If yes, find it.

**Fall 2018 Problem 5.** Solve the following questions:

(a) If $F$ is a field, prove that $F[x]/(f(x))$ is a field if and only if $f(x)$ is irreducible over $F$.

(b) Show that $f(x) = x^2 + 2x + 2$ is irreducible in $\mathbb{Q}[x]$, and find the inverse of $1 + x$ in $\mathbb{Q}[x]/(f(x))$.

**Fall 2017 Problem 4.** Find all maximal ideals in the ring $\mathbb{F}_7[x]/(x^2+1)$ and the ring $\mathbb{F}_7[x]/(x^3+1)$.

**Fall 2016 Problem 4.** Let $I, J \subseteq R$ be ideals in a principle ideal domain $R$. Prove that $I + J = R$ iff $IJ = I \cap J$.

**Fall 2014 Problem 4.** List all ideals of $\mathbb{F}_p[x]/(x^2 + x - 6)$ when

(a) $p = 7$
(b) $p = 5$.

**Fall 2015 Problem 4.** Let $I$ denote the ideal in the ring $\mathbb{Z}[x]$ generated by 5 and $x^3 + x + 1$. Is $I$ a prime ideal?