**Fall 2012 Problem 1:**

(1) Can a vector space over an infinite field be a finite union
$$V = \cup_{i=1}^{k} V_i$$
where for each $i$, $V_i \neq V$?

(2) Can the group $\mathbb{Z}^2 = \mathbb{Z} \times \mathbb{Z}$ be a union of fnitely many proper subgroups?

**Spring 2013 Problem 1:** Let $A$ be a *Boolean ring*, i.e., $a^a = a$ for all $a \in A$. Show that the ring $A$ is commutative.

**Fall 2012 Problem 4:** Let $R$ be a commutative ring and $I$ an ideal in $R$. Prove or disprove: The set $\sqrt{I} := \{a \in R : \exists n \in \mathbb{N}, n > 0, a^n \in I\}$ is an ideal.

**Spring 2016 Problem 6:** Find the smallest integer $c \geq 0$ for which

$$R_c := \mathbb{Z}[n]/(c, x^2 - 2)$$

is

(1) a domaiin
(2) a field.

**Spring 2014 Problem 5.** Let $V$ be a linear representation of the group $G$ over the field $\mathbb{Q}[\sqrt{2}]$, and let $\chi : G \to \mathbb{Q}[\sqrt{2}]$ be its character. Then $V$ is also a vector space $V_{\mathbb{Q}}$ over $\mathbb{Q}$, and is again a linear representation of $G$. Express its character $\chi_{\mathbb{Q}}$ in term of the original character $\chi$.

**Spring 2015 Problem 3.** Show that if $G$ is an infinite simple group then every proper subgroup has infinitely many conjugates. Use this to conclude that $G$ has an infinite automorphism group.

**Spring 2012 Problem 5.** Find the number of field homomorphisms $\phi : \mathbb{Q}[2^{\frac{1}{3}}] \to \mathbb{C}$.

**Spring 2014 Problem 1.** Find the smallest order of a group which is not cyclic and not isomorphic to a subgroup of the symmetric group $\Sigma_5$.