MAT 249, 2021, Day 7, Rings

(1) **Fall 2017 Problem 3.** Let $M_n(\mathbb{R})$ denote the ring of $n \times n$ matrices over $\mathbb{R}$, and consider a (possibly non-unital) ring homomorphsm $f : M_{n+1}(\mathbb{R}) \to M_n(\mathbb{R})$. Can f be nonzero?

(2) **Fall 2014 Problem 6.** If R is a commutative ring with identity, and S a multiplicative subset, then every ideal J of $S^{-1}R$ is of the form $S^{-1}I$ for some ideal I of R. Is I uniquely determined by J? Why or why not?

(3) **Fall 2011 Problem 1.** Show that there is no commutative ring with identity whose additive group is isomorphic to $\mathbb{Q}/\mathbb{Z}$.

(4) **Fall 2011 Problem 6.** Prove or disprove: $(\mathbb{Z}/35\mathbb{Z})^\times \cong (\mathbb{Z}/39\mathbb{Z})^\times \cong (\mathbb{Z}/45\mathbb{Z})^\times \cong (\mathbb{Z}/70\mathbb{Z})^\times \cong (\mathbb{Z}/78\mathbb{Z})^\times \cong (\mathbb{Z}/90\mathbb{Z})^\times$. Here $\cong (\mathbb{Z}/n\mathbb{Z})^\times$ is the group of units in $(\mathbb{Z}/n\mathbb{Z})$.

(5) **Fall 2010 Problem 5.** Let R be a ring with identity. Recall that $x \in R$ is called *nilpotent* if $x^n = 0$ for some n. Prove that if x is nilpotent, then $1 + x$ is invertible.

(6) **Winter 2008 Problem 3.** Recall that if R is a ring, an R-module M is projective means: If $f : A \to B$ is a homomorphism between two other R-modules and if $g : M \to B$ is a homomorphism, then there is always a solution $h : M \to A$ to the equation $g = fh$. Prove that among $\mathbb{Z}$-modules only the cyclic modules $\mathbb{Z}/n$ which is projective is $\mathbb{Z}/0 = \mathbb{Z}$.

(7) **Winter 2008 Problem 4.** Prove that $M_n(\mathbb{C})$ the algebra of $n \times n$ complex matrices, has no non-trival two-sided ideals.