1. Find the following derivatives or multiple derivatives assuming that $x > 0$:

(a) $\frac{d}{dx} \left[ \frac{5x+1}{2\sqrt{x}} \right]$

ANS: $\frac{5}{4}x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{5}{2}}$

(b) $\frac{d}{dx} \left[ \frac{\cos(x)}{2+\sin(x)} \right]$

ANS: $-\frac{\sin(x)[2+\sin(x)]-\cos^2(x)}{[2+\sin(x)]^2}$

(c) $\frac{d}{dx} \left[ \sin \left( \frac{x}{\sqrt{x+1}} \right) \right]$

ANS: $\cos \left( \frac{x}{\sqrt{x+1}} \right) \left( \frac{(x+1)^{-\frac{1}{2}}-x^{-\frac{3}{2}}}{x+1} \right)$

(d) $\frac{d}{dx} \left[ e^x + e^x + xe^x + x^2 \right]$

ANS: $0 + e^x + xe^{x-1} + x^2(\ln(x) + 1)$

(e) $\frac{d^2}{dx^2} \left[ \frac{5x+1}{2\sqrt{x}} \right]$

ANS: $-\frac{5}{8}x^{-\frac{5}{2}} + \frac{3}{8}x^{-\frac{7}{2}}$

(f) $\frac{d^4}{dx^4} \left[ 3\sin(2x) \right]$

ANS: $48\sin(2x)$

2. Find the following limit. (Do not give a decimal approximation.) $\lim_{h \to 0} \left[ \frac{3^h+2}{h} \right]$

ANS: $\lim_{h \to 0} \left[ \frac{3^h+2}{h} \right]$ is the limit $f'(3)$ if $f(x) = 3^x$ so $f'(x) = 3^x \ln(3)$ and the limit is $3^3 \ln(3)$.

3. Consider the curve $y = 2 \sin(\pi x - y)$. Find an equation for the line tangent to the curve at the point on the curve $(1, 0)$.

ANS: $y' = 2\cos(\pi x - y)(\pi - y')$ so at $(1, 0)$ one has $y' = 2\cos(\pi)(\pi - y') = -2\pi + 2y'$ and $y' = 2\pi$ is the desired slope giving an equation: $y = 2\pi(x-1)$.

4. The distance of a roomba from a wall after $t$ minutes is $s(t) = \frac{25t + 5}{5t + 5}$ feet.

(a) Find the average velocity of the roomba during the first five minutes.

ANS: $\frac{\frac{25\cdot5 + 5}{5\cdot5 + 5} - \frac{25}{5 + 5}}{4} = -\frac{1}{2}$.

(b) Find a time at which the instantaneous velocity of the roomba equals the average velocity you found in part (a).

ANS: $s'(t) = -25(t + 5)^{-2}$ which is $-\frac{1}{2}$ if $t = \sqrt{50} - 5$. 
5. A screen is on a wall so that if you stand \( x \) meters from the wall the apparent height of the screen in radians is

\[ h(x) = \arccot \left( \frac{x}{2} \right) - \arccot(x) \]

( This happens if the screen is one meter tall with its lowest point one meter above eye level. )

How far from the wall should you stand to get the largest apparent height of the screen?

**ANS:** Where the apparent height is largest \( h'(x) = 0 \). Computing:

\[ h'(x) = \frac{\frac{1}{1+x^2}}{1 + \frac{1}{1+x^2}} = \frac{2-x^2}{(4+x^2)(1+x^2)} \]

which is zero if \( x = \sqrt{2} \) meters.

6. Consider the five curves on the next page.

(a) Identify the graph of an implicit equation which is not \( y = k(x) \) for any function \( k \).

   A

(b) Identify the pair which are graphs of \( y = f(x) \) and \( y = f^{-1}(x) \).

   B and C

(c) Identify the pair which are graphs of \( y = g(x) \) and \( y = g'(x) \).

   E and D

7. (EC) A tern (a diving bird) follows a flight path with height \( y = \sin(x^2) \) meters when it is above a point \( x \) meters from shore. The bird hits the water and dives at \( x = \sqrt{\pi} \) meters while moving with a horizontal velocity of 5 meters per second.

(a) Find the slope of the bird’s path when \( x = \sqrt{\pi} \).

   **ANS:** \( \frac{dy}{dx} = 2x \cos(x^2) = -2\sqrt{\pi} \) which has no units.

(b) Find the vertical velocity of the bird when \( x = \sqrt{\pi} \).

   **ANS:** \( \frac{dy}{dt} = 2x \cos(x^2) \frac{dx}{dt} = -10\sqrt{\pi} \) meters per second.