Problem 1 (10 points): For the function \( f(x) = 2x \) over the interval \([0, 3]\)

a. (2 points) Graph the function over \([0, 3]\)

b. (5 points) Find a formula for the upper sum obtained by dividing the interval \([0, 3]\) into \(n\) equal subinterval.

c. (3 points) Take a limit of this sum as \(n \to \infty\) to calculate the area under the curve over \([0, 3]\).

\[
f(x) = 2x
\]

Since \(f\) is increasing on \([0, 3]\) we use right endpoints to obtain upper sums. \(\Delta x = \frac{3-0}{n} = \frac{3}{n}\) and \(x_i = i\Delta x = \frac{3i}{n}\). So an upper sum is

\[
\sum_{i=1}^{n} 2x_i \left( \frac{3}{n} \right) = \sum_{i=1}^{n} \frac{6i}{n} \cdot \frac{3}{n} = \frac{18}{n^2} \sum_{i=1}^{n} i = \frac{18}{n^2} \cdot \frac{n(n+1)}{2} = \frac{9n^2 + 9n}{n^2}
\]

Thus,

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{6i}{n} \cdot \frac{3}{n} = \lim_{n \to \infty} \frac{9n^2 + 9n}{n^2} = \lim_{n \to \infty} \left( 9 + \frac{9}{n} \right) = 9.
\]

Problem 2 (10 points): \( f(x) = 3x^2 - 3 \) on \([0, 1]\)

a. (4 points) Graph the function.

b. (6 points) Find its average value over the given interval.

\[
\text{av}(f) = \left( \int_{0}^{1} (3x^2 - 3) \, dx \right) / (1-0)
\]

\[= 3 \int_{0}^{1} x^2 \, dx - \int_{0}^{1} 3 \, dx = 3 \left( \frac{1^3}{3} \right) - 3(1-0)
\]

\[= -2.
\]
Problem 1 (10 points): For the function $f(x) = 3x^2$ over the interval $[0, 1]$

a. (2 points) Graph the function over $[0, 1]$

b. (5 points) Find a formula for the upper sum obtained by dividing the interval $[0, 1]$ into $n$ equal subintervals.

c. (3 points) Take a limit of this sum as $n \to \infty$ to calculate the area under the curve over $[0, 1]$.

Since $f$ is increasing on $[0, 1]$, we use right endpoints to obtain upper sums. $\Delta x = \frac{1-0}{n} = \frac{1}{n}$ and $x_i = i \Delta x = \frac{i}{n}$. So an upper sum is

$$\sum_{i=1}^{n} 3x_i^2 \left( \frac{1}{n} \right) = \sum_{i=1}^{n} 3 \left( \frac{i}{n} \right)^2 \left( \frac{1}{n} \right) = \frac{3}{n^2} \sum_{i=1}^{n} i^2 = \frac{3}{n^2} \cdot \left( \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \frac{2n^3 + 3n^2 + n}{2n^3} = \frac{2 + \frac{3}{2} + \frac{1}{n}}{2}. \text{ Thus, } \lim_{n \to \infty} \sum_{i=1}^{n} 3x_i^2 \left( \frac{1}{n} \right)$$

$$= \lim_{n \to \infty} \left( \frac{2 + \frac{3}{2} + \frac{1}{n}}{2} \right) = \frac{3}{2} = 1.$$

Problem 2 (10 points): $f(t) = t^2 - t$ on $[-2, 1]$

a. (4 points) Graph the function.

b. (6 points) Find its average value over the given interval.

$$av(f) = \left( \frac{1}{1-(-2)} \right) \int_{-2}^{1} (t^2 - t) \, dt$$

$$= \frac{1}{3} \int_{-2}^{1} t^2 \, dt - \frac{1}{3} \int_{-2}^{1} t \, dt$$

$$= \frac{1}{3} \int_{0}^{1} t^2 \, dt - \frac{1}{3} \int_{0}^{-2} t^2 \, dt - \frac{1}{3} \left( \left( \frac{1^3}{3} \right) - \left( \frac{(-2)^3}{3} \right) \right)$$

$$= \frac{1}{3} \left( \frac{1^3}{3} \right) - \frac{1}{3} \left( \frac{(-2)^3}{3} \right) + \frac{1}{3} = \frac{3}{2}.$$
No calculators, no cell phones, no aids.

This is a weekly midterm; you are expected to do your own work, and to adhere to the UC Davis Code of Academic Conduct.

Please show all your work, and mark your answers clearly.

Please indicate clearly if you continue work on the back of page.

Please stop immediately when time is called.

**Problem 1** (10 points): For the function \( f(x) = 3x + 2x^2 \) over the interval \([0, 1]\)

a. (2 points) Graph the function over \([0, 1]\)

b. (5 points) Find a formula for the upper sum obtained by dividing the interval \([0, 1]\) into \(n\) equal subintervals.

c. (3 points) Take a limit of this sum as \(n \to \infty\) to calculate the area under the curve over \([0, 1]\).

\[ f(x) = 3x + 2x^2 \]

\[
\sum_{i=1}^{n} (3x_i + 2x_i^2) \frac{1}{n} = \sum_{i=1}^{n} \left( \frac{3(i/n) + 2\left(\frac{i}{n}\right)^2}{n} \right) \frac{1}{n} = \frac{3}{n^3} \sum_{i=1}^{n} i + \frac{2}{n^3} \sum_{i=1}^{n} i^2
\]

\[
= \frac{3}{n^3} \left( \frac{n(n+1)}{2} \right) + \frac{2}{n^3} \left( \frac{n(n+1)(2n+1)}{6} \right) = \frac{3n^2 + 3n}{2n^3} + \frac{2n^3 + 3n + 1}{3n^3}
\]

\[
= \frac{3 + \frac{1}{2} + \frac{2 + \frac{1}{3} + \frac{1}{3}}{3}}{2}. \text{ Thus, } \lim_{n \to \infty} \sum_{i=1}^{n} (3x_i + 2x_i^2) \frac{1}{n}
\]

\[
= \lim_{n \to \infty} \left[ \left( \frac{3 + \frac{1}{2}}{2} \right) + \left( \frac{2 + \frac{1}{3} + \frac{1}{3}}{3} \right) \right] = \frac{3}{2} + \frac{2}{3} = \frac{13}{6}.
\]

**Problem 2** (10 points): \( h(x) = -|x| \) on \([-1, 1]\)

a. (4 points) Graph the function.

b. (6 points) Find its average value over the given interval.

\[
\text{(a)} \quad \text{av}(h) = \left( \frac{1}{1 - (-1)} \right) \int_{-1}^{0} -|x| \, dx = \int_{-1}^{0} (-x) \, dx
\]

\[
= \int_{-1}^{0} x \, dx = \frac{0^2}{2} - \frac{(-1)^2}{2} = -\frac{1}{2}.
\]

\[
\text{(b)} \quad \text{av}(h) = \left( \frac{1}{1 - 0} \right) \int_{0}^{1} -|x| \, dx = -\int_{0}^{1} x \, dx
\]

\[
= -\left( \frac{1^2}{2} - \frac{0^2}{2} \right) = -\frac{1}{2}.
\]

\[
\text{(c)} \quad \text{av}(h) = \left( \frac{1}{1 - (-1)} \right) \int_{-1}^{1} -|x| \, dx
\]

\[
= \frac{1}{2} \left( \int_{-1}^{0} -|x| \, dx + \int_{0}^{1} -|x| \, dx \right)
\]

\[
= \frac{1}{2} \left( \frac{1}{2} + \left( -\frac{1}{2} \right) \right) = -\frac{1}{2} \text{ (see parts (a) and (b) above).}
\]