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6.6 WORK

1. The force required to stretch the spring from its natural length of 2 m to a length of 5 m is $F(x) = kx$. The work done by $F$ is $W = \int_0^5 F(x) \, dx = k \int_0^5 x \, dx = \frac{1}{2} \left[ \frac{k}{2} \right]_0^5 = \frac{25k}{2}$. This work is equal to $1800 \, J \Rightarrow \frac{5}{2}k = 1800 \Rightarrow k = 400 \, N/m$

3. We find the force constant from Hooke's law: $F = kx$. A force of 2 N stretches the spring to the zero 0.02 m \Rightarrow 2 = k \cdot (0.02) \Rightarrow k = 100 \, N/m$. The force of 4 N will stretch the rubber band y m, where $F = ky \Rightarrow y = \frac{4}{k} \Rightarrow y = \frac{4N}{100N/m} \Rightarrow y = 0.04 \, m = 4 \, cm$. The work done to stretch the rubber band 0.04 m is $W = \int_0^{0.04} kx \, dx = 100 \int_0^{0.04} x \, dx = 100 \left[ \frac{x^2}{2} \right]_0^{0.04} = \frac{100\times(0.04)^2}{2} = 0.08 \, J$

7. The length required to haul up the rope is equal to the rope's weight, which varies steadily and is proportional to $x$, the length of the rope still hanging: $F(x) = 0.624x$. The work done is: $W = \int_0^x F(x) \, dx = \int_0^{0.624x} \, dx = 0.624 \left[ \frac{x^2}{2} \right]_0^{0.624x} = 780 \, J$

19. The typical slab between the planes at $y$ and and $y + \Delta y$ has a volume of $\Delta V = \pi (\text{radius})^2 (\text{thickness}) = \pi \left( \frac{3y}{2} \right)^2 \Delta y = \pi \cdot 100 \times \Delta y \, ft^3$. The force $F$ required to lift the slab is equal to its weight: $F = 51.2 \Delta V = 51.2 \cdot 100\pi \Delta y \, lb \Rightarrow F = 5120\pi \Delta y \, lb$. The distance through which $F$ must act is about $(30 - y) \, ft$. The work it takes to lift all the kerosene is approximately $W \approx \sum_{0}^{30} \Delta W = \sum_{0}^{30} 5120\pi(30 - y) \, dy \cdot \, lb$ which is a Riemann sum. The work to pump the tank dry is the limit of these sums: $W = \int_0^{30} 5120\pi(30 - y) \, dy = 5120\pi \left[ 30y - \frac{y^2}{2} \right]_0^{30} = 5120\pi \left( \frac{900}{2} \right) = (5120)(450\pi) \approx 7,238,229.48 \, ft \cdot lb$

21. (a) Follow all the steps of Example 5 but make the substitution of $\frac{64.5\pi}{16} \, lb$ for $\frac{57\pi}{4} \, lb$. Then,

$$W = \int_0^{\frac{64.5\pi}{4}} (10 - y)y^2 \, dy = \frac{64.5\pi}{4} \left[ \frac{10y^3}{3} - \frac{y^4}{4} \right]_0^{\frac{64.5\pi}{4}} = \frac{64.5\pi}{4} \left( \frac{10\times(81)}{3} - \frac{81}{4} \right) = \frac{64.5\pi}{4} \left( 270 - 2 \right) = 21.5\pi \cdot 83 \approx 34,582.65 \, ft \cdot lb$$

(b) Exactly as done in Example 5 but change the distance through which $F$ acts to distance $\approx (13 - y) \, ft$. Then

$$W = \int_0^{\frac{57\pi}{4}} (13 - y)y^2 \, dy = \frac{57\pi}{4} \left[ \frac{13y^3}{3} - \frac{y^4}{4} \right]_0^{\frac{57\pi}{4}} = \frac{57\pi}{4} \left( \frac{13\times(81)}{3} - \frac{81}{4} \right) = \frac{57\pi}{4} \left( 121 - 2 \right) = 57\pi \cdot 8.7
$$

$$= (19\pi) (8.7) (7)(2) \approx 53.482.5 \, ft \cdot lb$$

28. weight = 1.6 oz = 0.1 lb \Rightarrow m = \frac{0.1 \, lb}{32 \, ft/sec^2} = \frac{1}{320} \, slugs; W = \left( \frac{1}{2} \right) \left( \frac{1}{320} \, slugs \right) (280 \, ft/sec)^2 = 122.5 \, ft \cdot lb$

31. weight = 6.5 oz = \frac{6.5}{16} \, lb \Rightarrow m = \frac{6.5}{16} (16)(32) \, slugs; W = \left( \frac{1}{2} \right) \left( \frac{6.5}{16} \, slugs \right) (132 \, ft/sec)^2 \approx 110.6 \, ft \cdot lb$

35. We imagine the milkshake divided into thin slabs by planes perpendicular to the y-axis at the points of a partition of the interval $[0, 7]$. The typical slab between the planes at $y$ and $y + \Delta y$ has a volume of about $\Delta V = \pi (\text{radius})^2 (\text{thickness}) = \pi \left( \frac{y + 17.5}{14} \right)^2 \Delta y \, in^3$. The force $F(y)$ required to lift this slab is equal to its weight: $F(y) = \frac{4}{9} \Delta V = \frac{4}{9} \left( \frac{y + 17.5}{14} \right)^2 \Delta y \, oz$. The distance through which $F(y)$ must act to lift this slab to the level of 1 inch above the top is about $(8 - y) \, in$. The work done lifting the slab is about

$$\Delta W = \frac{4}{9} \left( \frac{y + 17.5}{14} \right)^2 \Delta y \, in \cdot oz$$. The work done lifting all the slabs from $y = 0$ to $y = 7$ is approximately $W = \sum_{0}^{7} \frac{4}{9} \left( 8 + 17.5 \right)^2 \Delta y \, in \cdot oz$ which is a Riemann sum. The work is the limit of these sums as the norm of the partition goes to zero: $W = \int_0^7 \frac{4}{9} \left( 8 + 17.5 \right)^2 (8 - y) \, dy$

$$= \frac{4}{9} \int_0^7 (2450 - 26.25y - 27y^2 - y^3) \, dy = \frac{4}{9} \left[ \frac{2450}{2} - \frac{9y^2}{4} - 9y^3 - \frac{25}{2}y^2 + 2450y \right]_0^7$$

$$= \frac{4}{9} \left[ - \frac{71}{4} - 2y^3 \right]_0^7 = 91.32 \, in \cdot oz$
6.7 FLUID PRESSURES AND FORCES

1. To find the width of the plate at a typical depth \( y \), we first find an equation for the line of the plate's right-hand edge: \( y = x - 5 \). If we let \( x \) denote the width of the right-hand half of the triangle at depth \( y \), then \( x = 5 + y \) and the total width is \( L(y) = 2x = 2(5 + y) \). The depth of the strip is \( (-y) \). The force exerted by the water against one side of the plate is therefore \( F = \int_{-5}^{5} w(-y) \cdot L(y) \, dy = \int_{-5}^{5} 62.4 \cdot (-y) \cdot 2(5 + y) \, dy \)

\[
= 124.8 \int_{-5}^{5} (-y^2 - 2y^3) \, dy = 124.8 \left[ -\frac{2}{3} y^3 - \frac{1}{4} y^4 \right]_{-5}^{5} = 124.8 \left[ (-\frac{2}{3} \cdot 4 + \frac{1}{4} \cdot 8) - (-\frac{2}{3} \cdot 25 + \frac{1}{4} \cdot 125) \right]
\]

\[
= (124.8) \left( \frac{105}{2} - \frac{117}{4} \right) = (124.8) \left( \frac{315 - 234}{6} \right) = 1684.8 \text{ lb}
\]

9. Using the coordinate system given in the accompanying figure, we see that the right-hand edge is \( x = \sqrt{1 - y^2} \), so the total width is \( L(y) = \frac{2 \sqrt{1 - y^2}}{2} \) and the depth of the strip is \( (-y) \). The force exerted by the water is therefore \( F = \int_{-1}^{1} w \cdot (-y) \cdot 2\sqrt{1 - y^2} \, dy \)

\[
= 62.4 \int_{-1}^{1} \sqrt{1 - y^2} \, d(1 - y^2) = 62.4 \left[ \frac{2}{3} (1 - y^2)^{3/2} \right]_{-1}^{0} = (62.4) \left( \frac{1}{2} \right) (1 - 0) = 416 \text{ lb}
\]

13. Suppose that \( h \) is the maximum height. Using the coordinate system given in the text, we find an equation for the line of the end plate's right-hand edge is \( y = \frac{3}{2} x \Rightarrow \frac{x}{2} = \frac{3}{5} y \). The total width is \( L(y) = 2x = \frac{6}{5} y \) and the depth of the typical horizontal strip at level \( y \) is \( (h - y) \). Then the force is \( F = \int_{y}^{h} w(h - y) L(y) \, dy = F_{\text{max}} \), where \( F_{\text{max}} = 6667 \text{ lb} \). Hence, \( F_{\text{max}} = w \int_{y}^{h} (h - y) \cdot \frac{3}{2} y \, dy = (62.4) \left( \frac{3}{2} \right) \int_{y}^{h} (hy - y^2) \, dy \)

\[
= (62.4) \left( \frac{3}{2} \right) \left[ \frac{hy^2}{2} - \frac{y^3}{3} \right]_{y}^{h} = (62.4) \left( \frac{3}{2} \right) \left( \frac{h^3}{2} - \frac{y^3}{3} \right) = (62.4) \left( \frac{3}{2} \right) \left( \frac{h^3}{2} \right) = (10.4) \left( \frac{3}{2} \right) h^3 \Rightarrow h = \sqrt[3]{\frac{4}{15}} \left( \frac{F_{\text{max}}^{1/3}}{10.4} \right)
\]

15. The pressure at level \( y \) is \( p(y) = w \cdot y \Rightarrow \) the average pressure is \( \bar{p} = \frac{1}{b} \int_{y}^{b} p(y) \, dy = \frac{1}{b} \int_{y}^{b} w \cdot y \, dy = \frac{1}{b} w \left[ \frac{y^2}{2} \right]_{y}^{b} = \left( \frac{y}{b} \right) \left( \frac{b^2}{2} \right) = \frac{y}{b} \cdot \frac{b^2}{2} \). This is the pressure at level \( \frac{y}{b} \), which is the pressure at the middle of the plate.

21. (a) An equation of the right-hand edge is \( y = \frac{3}{2} x \Rightarrow \frac{x}{2} = \frac{3}{5} y \) and \( L(y) = 2x = \frac{6}{5} y \). The depth of the strip is \( (3 - y) \Rightarrow F = \int_{0}^{y} w(3 - y) L(y) \, dy = \int_{0}^{y} (62.4)(3 - y) \left( \frac{3}{5} y \right) \, dy = (62.4) \cdot \left( \frac{3}{5} \right) \int_{0}^{y} (3y - y^2) \, dy \)

\[
= (62.4) \left( \frac{3}{5} \right) \left[ \frac{3y^2}{2} - \frac{y^3}{3} \right]_{0}^{y} = (62.4) \left( \frac{3}{5} \right) \left[ \frac{3y^2}{2} - \frac{y^3}{3} \right]_{0}^{y} = (62.4) \left( \frac{3}{5} \right) \left( \frac{3y^2}{2} \right) = 374.4 \text{ lb}
\]

(b) We want to find a new water level \( Y \) such that \( F_Y = \frac{1}{2} (374.4) = 187.2 \text{ lb} \). The new depth of the strip is \( (Y - y) \), and \( Y \) is the new upper limit of integration. Thus, \( F_Y = \int_{0}^{Y} w(Y - y) L(y) \, dy \)

\[
= 62.4 \int_{0}^{Y} (Y - y) \left( \frac{3}{5} y \right) \, dy = (62.4) \left( \frac{3}{5} \right) \int_{0}^{Y} (Yy - y^2) \, dy = (62.4) \left( \frac{3}{5} \right) \left[ \frac{Y^2}{2} - \frac{y^3}{3} \right]_{0}^{Y} = (62.4) \left( \frac{3}{5} \right) \left( \frac{Y^2}{2} - \frac{Y^3}{3} \right)
\]

\[
= (62.4) \left( \frac{3}{5} \right) Y^3 \Rightarrow \text{Therefore } Y^3 = \frac{99(187.2)}{2(62.4)} \Rightarrow Y = \sqrt[3]{\frac{99(187.2)}{2(62.4)}} \approx 3 \text{ ft} \approx 3.811 \text{ ft} \approx 0.6189 \text{ in.} \approx 7.5 \text{ in.} \text{ to the nearest half inch.}
\]

(c) No, it does not matter how long the trough is. The fluid pressure and the resulting force depend only on depth of the water.