No calculators, no cell phones, no aids.
This is a weekly midterm; you are expected to do your own work, and to adhere to the UC Davis Code of Academic Conduct.
Please show all your work, and mark your answers clearly.
Please indicate clearly if you continue work on the back of page.
Please stop immediately when time is called.

**Problem 1** (10 points): Solve the initial value problem:
\[
\frac{dy}{dx} = 4x \left(x^2 + 8\right)^{-\frac{1}{3}}, \quad y(0) = 0
\]

Let \( u = x^2 + 8 \Rightarrow du = 2x \, dx \Rightarrow 2 \, du = 4x \, dx \)
\[ y = \int 4x \left(x^2 + 8\right)^{-1/3} \, dx = \int u^{-1/3} \, (2 \, du) = 2 \left(\frac{2}{3} u^{2/3}\right) + C = 3u^{2/3} + C = 3(x^2 + 8)^{2/3} + C; \]
\[ y = 0 \text{ when } x = 0 \Rightarrow 0 = 3(8)^{2/3} + C \Rightarrow C = -12 \Rightarrow y = 3(x^2 + 8)^{2/3} - 12 \]

**Problem 2** (10 points + 4 extra points):
Simplify the expression
\[
\frac{d}{dx} \int_{\gamma(x)}^{\omega(x)} f(u) \, du
\]
You must use complete arguments in full sentences. Not just give formulas step by step.
**Hint:** You may need to use definition of derivative, definition of integral, Chain Rule, the fact that integral is antiderivative, use properties of integral (its upper/lower limits, etc.)

(2.a) (4 extra points) Show that \( \frac{d}{dw} \int_{a}^{w} f(u) \, du = f(w) \)
(2.b) (5 points) Show that \( \frac{d}{dx} \int_{a}^{w(x)} f(u) \, du = f(w(x)) \omega'(x) \)
(2.c) (5 points) From what we get in parts (a) and (b), simplify \( \frac{d}{dx} \int_{\gamma(x)}^{\omega(x)} f(u) \, du \)
Problem 1 (10 points): Evaluate the integral:
\[ \int r^4 \left( 7 - \frac{r^4}{10} \right)^3 \, dr \]

Let \( u = 7 - \frac{r^4}{10} \Rightarrow \frac{du}{dr} = -\frac{1}{10} r^3 \)
\[ \Rightarrow -\frac{1}{10} \, du = r^3 \, dr \]
\[ \int r^4 \left( 7 - \frac{r^4}{10} \right)^3 \, dr = \int u^3 \cdot (-2 \, du) = -2 \int u^3 \, du = -2 \left( \frac{u^4}{4} \right) + C = -\frac{1}{2} \left( 7 - \frac{r^4}{10} \right)^4 + C \]

Problem 2 (10 points + 4 extra points):
Simplify the expression
\[ \frac{d}{dx} \int_{V(x)}^{w(x)} f(u) \, du \]

You must use complete arguments in full sentences. Not just give formulas step by step.

Hint: You may need to use definition of derivative, definition of integral, Chain Rule, the fact that integral is antiderivative, use properties of integral (its upper/lower limits, etc.)

(2.a) (4 extra points) Show that \( \frac{d}{dw} \int_{a}^{w} f(u) \, du = f(w) \)

(2.b) (5 points)
Show that \( \frac{d}{dx} \int_{a}^{w(x)} f(u) \, du = f(w(x)) \cdot w'(x) \)

(2.c) (5 points) From what we get in parts (a) and (b), simplify \( \frac{d}{dx} \int_{V(x)}^{w(x)} f(u) \, du \)
Problem 1 (10 points): Evaluate the integral:
\[ \int x^{1/3} \sin\left(x^{4/3} - 8\right) \, dx \]

Let \( u = x^{4/3} - 8 \Rightarrow du = \frac{4}{3} x^{1/3} \, dx \Rightarrow \frac{3}{4} du = x^{1/3} \, dx \)
\[
\int x^{1/3} \sin\left(x^{4/3} - 8\right) \, dx = \int \sin u \left(\frac{3}{4} du\right) = \frac{3}{4} \int \sin u \, du = \frac{3}{4} (-\cos u) + C = -\frac{3}{4} \cos\left(x^{4/3} - 8\right) + C
\]

Problem 2 (10 points + 4 extra points):
Simplify the expression
\[ \frac{d}{dx} \int_{v(x)}^{w(x)} f(u) \, du \]
You must use complete arguments in full sentences. Not just give formulas step by step. 
**Hint:** You may need to use definition of derivative, definition of integral, Chain Rule, the fact that integral is antiderivative, use properties of integral (its upper/lower limits, etc.)

(2.a) (4 extra points) Show that \( \frac{d}{du} \int_{a}^{w} f(u) \, du = f(w) \)

(2.b) (5 points)
Show that \( \frac{d}{dx} \int_{a}^{w(x)} f(u) \, du = f(w(x)) w'(x) \)

(2.c) (5 points) From what we get in parts (a) and (b), simplify \( \frac{d}{dx} \int_{v(x)}^{w(x)} f(u) \, du \)
(2. a) See the proof of FTC part 1 in the textbook on pages 387 - 388.

(2. b) Show that \[ \frac{d}{dx} \int_a^{w(x)} f(u) du = f(w(x)) w'(x) \]

From part (2.a), we have \[ \frac{d}{dw} \int_a^w f(u) du = f(w) \].

Using chain rule, we have

\[ \frac{d}{dx} \int_a^{w(x)} f(u) du = \frac{dw}{dx} \frac{d}{dw} \int_a^w f(u) du \]

\[ = f(w(x)) w'(x) \]

(2. c) \[ \frac{d}{dx} \int_{v(x)}^{w(x)} f(u) du \]

\[ = \frac{d}{dx} \left( \int_{v(x)}^a f(u) du + \int_a^{w(x)} f(u) du \right) \]

\[ = \frac{d}{dx} \left( \int_a^{w(x)} f(u) du - \int_a^{v(x)} f(u) du \right) \]

\[ = \frac{d}{dx} \int_a^{w(x)} f(u) du - \frac{d}{dx} \int_a^{v(x)} f(u) du \]

\[ = f(w(x)) w'(x) - f(v(x)) v'(x) \]