Winter 2006
MAT 21B
MAT 21B Section A
Professor Joseph Biello
Section
Time
TA: David

PRINT Name: ____________________________
(Last Name) (First Name)

- No calculators, no cell phones, no aids.
- This is a weekly midterm; you are expected to do your own work, and to adhere to the UC Davis Code of Academic Conduct.
- Please show all your work, and mark your answers clearly.
- Please indicate clearly if you continue work on the back of page.
- Please stop immediately when time is called.

**Problem 1 (7 points each):**
Find the volume of the solids generated by revolving the regions bounded by the curves and lines about the x-axis using disk, washer or shell methods:

(a) $y = x^3, \quad y = 0, \quad x = 2$

(b) $x = y^2, \quad x = -y, \quad y = 2, \quad y \geq 0$

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20. $R(x) = x^3 \Rightarrow V = \int_0^2 \pi (R(x))^2 \, dx = \pi \int_0^2 (x^3)^2 \, dx$

$$= \pi \int_0^2 x^6 \, dx = \pi \left[ \frac{x^7}{7} \right]_0^2 = \frac{128\pi}{7}$$

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16. $c = 0, \, d = 2$:

$V = \int_2^4 2\pi \left( \text{shell radius} \right) \left( \text{shell height} \right) \, dy = \int_0^2 2\pi y \left[ y^2 - (-y) \right] \, dy$

$$= 2\pi \int_0^2 (y^3 + y^2) \, dy = 2\pi \left[ \frac{y^4}{4} + \frac{y^3}{3} \right]_0^2 = 16\pi \left( \frac{1}{4} + \frac{1}{3} \right)$$

$$= 16\pi \left( \frac{7}{12} \right) = \frac{28\pi}{3}$$
Problem 2 (6 points): Find the length of the curve.

\[ X = t^{2/2}, \quad Y = (2t + 1)^{3/2}/3 \quad 0 \leq t \leq 4 \]

4. \( \frac{dx}{dt} = t \) and \( \frac{dy}{dt} = (2t + 1)^{1/2} \) \( \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{t^2 + (2t + 1)} = \sqrt{(t + 1)^2} = |t + 1| = t + 1 \) since \( 0 \leq t \leq 4 \)

\( \Rightarrow \text{Length} = \int_0^4 (t + 1) \, dt = \left[ \frac{t^2}{2} + t \right]_0^4 = (8 + 4) = 12 \)
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Problem 1 (7 points each):
Find the volume of the solids generated by revolving the regions bounded by the curves and lines about the x-axis using disk, washer or shell methods:

(a) \( y = x - x^2, \quad y = 0 \)

(b) \( y = x, \quad y = 2x, \quad y = 2 \)

22. \( R(x) = x - x^2 \Rightarrow V = \int_0^1 \pi[R(x)]^2 \, dx = \pi \int_0^1 (x - x^2)^2 \, dx \)
   \[ = \pi \int_0^1 (x^2 - 2x^3 + x^4) \, dx = \pi \left[ \frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right]_0^1 \]
   \[ = \pi \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = \frac{\pi}{30} (10 - 15 + 6) = \frac{\pi}{30} \]

20. \( c = 0, \, d = 2; \)

\[ V = \int_0^2 \pi \left( \frac{\text{shell}}{\text{radius}} \right) \left( \frac{\text{shell}}{\text{height}} \right) dy = \int_0^2 2\pi y (y - \frac{2}{3}) dy \]
\[ = 2\pi \int_0^2 y^2 dy = \frac{\pi}{3} \left[ y^3 \right]_0^2 = \frac{8\pi}{3} \]
Problem 2 (6 points): Find the length of the curve.
\[ x = 8 \cos t + 8 t \sin t, \quad y = 8 \sin t - 8 t \cos t, \quad 0 \leq t \leq \pi / 2 \]

6. \[ \frac{dx}{dt} = 8 \cos t \text{ and } \frac{dy}{dt} = 8 t \sin t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(8 \cos t)^2 + (8 t \sin t)^2} = \sqrt{64 t^2 \cos^2 t + 64 t^2 \sin^2 t} \]

\[ = |8t| = 8t \text{ since } 0 \leq t \leq \frac{\pi}{2} \Rightarrow \text{Length} = \int_0^{\pi/2} 8 t \, dt = \left[4t^2\right]_0^{\pi/2} = \pi^2 \]
Problem 1 (7 points each):
Find the volume of the solids generated by revolving the regions bounded by the curves and lines about the x-axis using disk, washer or shell methods:

(a) \( y = 5 \cos x, y = 0, x = -\pi/4, x = \pi/4 \)
(b) \( y = \sqrt{x}, y = 0, y = 2 - x \)

24. \( R(x) = \sec x \Rightarrow V = \int_{-\pi/4}^{\pi/4} \pi (R(x))^2 \, dx = \pi \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx \)
   \[ = \pi [\tan x]_{-\pi/4}^{\pi/4} = \pi [1 - (-1)] = 2\pi \]

22. \( c = 0, d = 1; \)
   \[ V = \int_{-1}^{1} 2\pi \left( \text{shell radius} \right) \left( \text{shell height} \right) \, dy = \int_{0}^{1} 2\pi y [(2 - y) - y^2] \, dy \]
   \[ = 2\pi \int_{0}^{1} (2y - y^2 - y^3) \, dy = 2\pi \left[ y^2 - \frac{y^3}{3} - \frac{y^4}{4} \right]_{0}^{1} \]
   \[ = 2\pi \left( 1 - \frac{1}{3} - \frac{1}{4} \right) = \frac{\pi}{6} (12 - 4 - 3) = \frac{5\pi}{6} \]
Problem 2 (6 points): Find the length of the curve.

\[ x = e^t \cos t, \quad y = e^t \sin t, \quad 0 \leq t \leq \pi \]

8. \( \frac{dx}{dt} = e^t (\cos t - \sin t) \) and \( \frac{dy}{dt} = e^t (\sin t + \cos t) \)

\[ \Rightarrow \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} = \sqrt{\left[ e^t (\cos t - \sin t) \right]^2 + \left[ e^t (\sin t + \cos t) \right]^2} = \sqrt{2e^{2t}} \]

\[ = \sqrt{2} e^t \Rightarrow L = \int_0^\pi \sqrt{2} e^t \, dt = \left[ \sqrt{2} e^t \right]_0^\pi = \sqrt{2}(e^\pi - 1) \]