Problem 1 (10 points):
The conical tank in the figure is completely full with olive oil weighing 57 lb/ft\(^3\). How much work does it take to pump the oil to the level of the top of the tank?

The slab is a disk of area \(\pi x^2 = \pi \left(\frac{y}{2}\right)^2\), thickness \(\triangle y\), and height below the top of the tank \((10 - y)\). So the work to pump the oil in this slab, \(\triangle W\), is \(57(10 - y)\pi \left(\frac{y}{2}\right)^2\). The work to pump all the oil to the top of the tank is

\[
W = \int_0^{10} \frac{57\pi}{4} (10y^2 - y^3)dy = \frac{57\pi}{4} \left[ \frac{10y^3}{3} - \frac{y^4}{4} \right]_0^{10} = 11,875\pi \text{ ft} \cdot \text{lb} \approx 37,306 \text{ ft} \cdot \text{lb}.
\]
Problem 2 (10 points): The face of a dam is a rectangle, ABCD, of dimensions AB = CD = 100 ft, AD = BC = 26 ft. Instead of being vertical, the plane ABCD is inclined as indicated in the accompanying figure, so that the top of the dam is 24 ft higher than the bottom. Find the force due to water pressure on the dam when the surface of the water is level with the top of the dam.

22. The area of a strip of the face of height \( \Delta y \) and parallel to the base is \( 100 \left( \frac{26}{24} \right) \cdot \Delta y \), where the factor of \( \frac{26}{24} \) accounts for the inclination of the face of the dam. With the origin at the bottom of the dam, the force on the face is then:

\[
F = \int_0^{24} w(24 - y)(100)\left( \frac{26}{24} \right) dy = 6760 \left[ 24y - \frac{y^2}{2} \right]_0^{24} = 6760 \left( 24^2 - \frac{24^2}{2} \right) = 1,946,880 \text{ lb}.
\]

Another write-up:

By similar \( \Delta 's \)

\[
\frac{26}{24} = \frac{L}{dy} \quad \Rightarrow \quad L = \left( \frac{26}{24} \right) dy
\]

Water pressure on this strip is

\[
\approx (\text{area})(\text{depth})(\text{density})
\]

\[
= (\text{length})(\text{width})(\text{depth})(\text{density})
\]

\[
= (100) \left( \frac{26}{24} \cdot dy \right)(24 - y) (62.4)
\]

\[
= 6240 \left( \frac{13}{12} \right)(24 - y) \cdot dy
\]

So the total force \( F = \int_0^{24} 6240 \left( \frac{13}{12} \right)(24 - y) \cdot dy \)

\[
= 6240 \left( \frac{13}{12} \right)(24 - \frac{24^2}{2}) = \left[ \frac{6240 \left( \frac{13}{12} \right)(24 - \frac{24^2}{2})}{24} \right]_0^{24}
\]
Problem 1 (10 points):
The conical tank in the figure is half full (1/2 of the volume of the cone, not the height of the cone) with olive oil weighing 57 lb/ft³. How much work does it take to pump the oil to the level of the top of the tank?

Each slab of oil to be pumped to a height of 10 ft.
So the work to pump a slab is \( (10 - y) \pi \left( \frac{y}{2} \right)^2 \) dy and since the tank is half full and the volume of the original cone is \( V = \frac{1}{3} \pi r^2 h \)
\[ = \frac{1}{6} \pi (5^3) (10) = \frac{250 \pi}{3} \text{ ft}^3 \]
half the volume \( = \frac{250 \pi}{6} \text{ ft}^3 \)

and with half the volume the cone is filled to a height \( y \),

\[ \frac{1}{2} V = \frac{250 \pi}{6} = \frac{1}{3} \pi \left( \frac{y}{2} \right)^2 y \Rightarrow y = \sqrt[3]{\frac{2500}{\pi}} \text{ ft} \left( 500^{\frac{1}{3}} \text{ ft} \right) \]

So:
\[ W = \int_0^{\sqrt[3]{\frac{2500}{\pi}}} 57 (10 - y) \pi \left( \frac{y}{2} \right)^2 \text{ dy} = \int_0^{\sqrt[3]{\frac{2500}{\pi}}} \frac{57 \pi}{4} (10y^2 - y^3) \text{ dy} = \frac{57 \pi}{4} \left[ 10 \cdot \frac{y^3}{3} - \frac{y^4}{4} \right]_0^{\sqrt[3]{\frac{2500}{\pi}}} \approx 30198 \text{ ft-lb} \]

Students get full credit if reach this point.
**Problem 2** (10 points): The face of a dam is a rectangle, ABCD, of dimensions AB = CD = 100 ft, AD = BC = 39 ft. Instead of being vertical, the plane ABCD is inclined as indicated in the accompanying figure, so that the top of the dam is 36 ft higher than the bottom. Find the force due to water pressure on the dam when the surface of the water is level with the top of the dam.

By similar \( \Delta \)'s

\[
\frac{39}{36} \frac{dy}{y} = \frac{L}{dy} \rightarrow L = \left( \frac{39}{36} \right) dy
\]

The water pressure on this strip is

\[
\approx (\text{area}) (\text{depth}) (\text{density})
\]

\[
= (\text{length}) (\text{width}) (\text{depth}) (\text{density})
\]

\[
= (100) \left( \frac{39}{36} \right) (36-y) (62.4)
\]

\[
= 6240 \left( \frac{13}{12} \right) (36-y) dy
\]

So the total force

\[
F = \int_0^{36} 6240 \left( \frac{13}{12} \right) (36-y) dy
\]

\[
= 6240 \left( \frac{13}{12} \right) \left( 36 \cdot \frac{y}{2} - \frac{y^2}{2} \right) \bigg|_0^{36} = \boxed{6240 \left( \frac{13}{12} \right) \left( 36^2 - \frac{36^2}{2} \right)}
\]

FULL CREDIT
Problem 1 (10 points): The conical tank in the figure is half full (1/2 of the volume of the cone, not the height of the cone) with olive oil weighing 57 lb/ft$^3$. How much work does it take to pump the oil to a level 4 ft above the top of the tank?

Each slab of oil is to be pumped to a height of 14 ft. So the work to pump a slab is $(14 - y)(\pi)(\frac{1}{2})^2$ and since the tank is half full and the volume of the original cone is $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (5^2)(10) = \frac{250\pi}{3}$ ft$^3$, half the volume is $\frac{250\pi}{6}$ ft$^3$, and with half the volume the cone is filled to a height $y$, $\frac{250\pi}{6} = \frac{1}{3} \pi \frac{5^2}{4} y \Rightarrow y = \sqrt{\frac{500}{\pi}}$ ft. So $W = \int_0^{\sqrt{\frac{500}{\pi}}} \frac{57\pi}{4} (14y^2 - y^3) \, dy$

$$= \frac{57\pi}{4} \left[ \frac{14}{4} \frac{1}{3} - \frac{y^4}{4} \right]_0^{\sqrt{\frac{500}{\pi}}} \approx 60,042 \text{ ft} \cdot \text{lb}.$$ 

Students get full credit if reach this point.
Problem 2 (10 points): The face of a dam is a rectangle, ABCD, of dimensions AB = CD = 100 ft, AD = BC = 15 ft. Instead of being vertical, the plane ABCD is inclined as indicated in the accompanying figure, so that the top of the dam is 12 ft higher than the bottom. Find the force due to water pressure on the dam when the surface of the water is level with the top of the dam.

By similar Δ's,
\[
\frac{15}{12} = \frac{L}{dy} \rightarrow L = \left(\frac{15}{12}\right) dy
\]

The water pressure on this strip is
\[
\approx \text{(area)} \times \text{(depth)} \times \text{(density)}
\]
\[
= \text{(length)} \times \text{(width)} \times \text{(depth)} \times \text{(density)}
\]
\[
= (100) \left(\frac{15}{12} \ dy\right) (12-y) (62.4)
\]
\[
= 6240 \left(\frac{5}{4}\right) (12-y) dy
\]

So the total force \( F = \int_0^{12} 6240 \left(\frac{5}{4}\right) (12-y) dy \)
\[
= 6240 \left(\frac{5}{4}\right) (12-y - \frac{y^2}{2})|_0^{12} = 6240 \left(\frac{5}{4}\right) \left(12 - 12 - \frac{12^2}{2}\right) = \boxed{6240 \left(\frac{5}{4}\right) \left(\frac{12^2 - 12^2}{2}\right)}
\]