WRITE YOUR NAME AND SECTION NUMBER ON THE FRONT COVER OF THE EXAM RIGHT NOW.

This is a closed note exam. No books, notes or other references are allowed.

No calculators are allowed.

No cell phones or other electronic devices are allowed.

Please write your answers directly on this exam.

Name: ________________________ ________________________

Section: ________________________ ________________________
Problem 1. (10 points) Find the limits of the following sequences. Justify your answers.

(a) \[ \lim_{n \to \infty} \frac{\sin(n)}{n} \]

(b) \[ \lim_{n \to \infty} \left( 1 + \frac{2}{n} \right)^n \]
Problem 2. (10 points) Find the values of the following series.

(a) \[ \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n \]

(b) \[ \sum_{n=1}^{\infty} \frac{1}{n^2 + n} \]
Problem 3. (20 points) Determine whether each of the following series converge or diverge. Give reasons for your answers.

(a) \[ \sum_{n=0}^{\infty} \frac{1}{n^2 + 2n + 1} \]

(b) \[ \sum_{n=0}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^n \]

(c) \[ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)} \]

(d) \[ \sum_{n=1}^{\infty} \frac{\ln(n)}{n^2} \]
Problem 4. (20 points) Find the radius of convergence and interval of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{n^{1/n}}{4^n} x^n \]

Justify your answer.
Problem 5. (20 points) (a) Find the Taylor series for the function \( f(x) = \frac{x^2}{2-x} \) around the point \( x_0 = 1 \).

(b) Find the interval of convergence of that Taylor series. Justify your answer.
Problem 6. (20 points) (a) Find the Maclaurin series for the function
\[ \frac{1}{1 + x^2}. \]
(b) Find the Maclaurin series for the function
\[ \arctan(x). \]
(c) Find the Maclaurin series for the function
\[ \arctan(x^2). \]