Problem One (5 points). Find the equation of the line through the point \((1, 1, 1)\) and parallel to the \(z\)-axis.

Solution: The vector \((0, 0, 1)\) is normal to the \(z\)-axis and hence normal to the plane \(P\) whose equation we wish to write. Since the plane goes through the point \((1, 1, 1)\), we know that if \((x, y, z)\) is a point on \(P\) then

\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix} \cdot \begin{pmatrix}
x - 1 \\
y - 1 \\
z - 1
\end{pmatrix} = 0.
\]

So

\[0(x - 1) + 0(y - 1) + 1(z - 1) = 0\]

or

\[z - 1 = 0\]

is an equation for the plane \(P\).

Problem Two (5 points). Write down an equation for the plane containing the points \((1, 1, -1)\), \((2, 0, 2)\) and \((0, -2, 1)\).

Solution: Let \(v_1 = (1, 1, -1)\), \(v_2 = (2, 0, 2)\), and \(v_3 = (0, -2, 1)\).

A normal \(N\) to the plane \(P\) whose equation we seek can be found as follows:

\[
N = (v_2 - v_1) \times (v_3 - v_1)
\]

\[
= \begin{pmatrix}
1 \\
-1 \\
3
\end{pmatrix} \times \begin{pmatrix}
-1 \\
-3 \\
2
\end{pmatrix}
\]

\[
= \begin{pmatrix}
7 \\
-5 \\
-4
\end{pmatrix}.
\]

If \((x, y, z)\) is a point on the plane \(P\), then we have

\[
\begin{pmatrix}
7 \\
-5 \\
-4
\end{pmatrix} \cdot \begin{pmatrix}
x - 1 \\
y - 1 \\
z + 1
\end{pmatrix} = 0
\]

or

\[7(x - 1) - 5(y - 1) - 4(z + 1) = 0.\]