Problem One (5 points). Show that the function
\[ f(x, y) = \frac{x^4}{x^4 + y^2} \]
has no limit as \((x, y) \to (0, 0)\).

Solution: If we approach along the line \(y = x\), we get the limit
\[ \lim_{x \to 0} \frac{x^4}{x^4 + x^2} = 0. \]
If we approach along the line \(y = x^2\), we get the limit
\[ \lim_{x \to 0} \frac{x^4}{x^4 + x^4} = 1/2. \]
The limit cannot exist since different limits are obtained by approaching \((0, 0)\) along different paths.

Problem Two (5 points). Let \(f(x, y) = \sin(xy)\). Find the derivatives
\[ \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial x^2}, \quad \text{and} \quad \frac{\partial^2 f}{\partial y \partial x}. \]

Solution: Since
\[ \frac{\partial f}{\partial y} = x \cos(xy) \]
we have:
\[ \frac{\partial^2 f}{\partial x \partial y} = \cos(xy) - xy \sin(xy). \]
Since
\[ \frac{\partial f}{\partial x} = y \cos(xy) \]
we have:
\[ \frac{\partial^2 f}{\partial x^2} = -y^2 \sin(xy) \]
and
\[ \frac{\partial^2 f}{\partial y \partial x} = \cos(xy) - xy \sin(xy). \]