Selected problem for HW 4

7.3.18.

\[
\frac{d}{dy} \arctan y = \frac{\frac{d}{dx} \tan x}{1 + \tan^2 x} \bigg|_{x=\arctan y} = \frac{1}{\sec^2 (\arctan y)} = \frac{1}{1 + y^2}
\]

7.6.6.

Without loss of generality, we assume that \( f'(x) > 0 \) \( \forall x \in (a, b) \). By MVT, we have \( f(b) > f(a) \). I claim that \( f \) is 1-1 and onto \( [f(a), f(b)] \). The "onto"ness is clear (by continuity of \( f \) and completeness of \( \mathbb{R} \)). To see the "1-1"ness, we assume otherwise for a contradiction. Suppose we have \( c, d \in [a, b] \) with \( f(c) = f(d) \). Then by Rolle's Thm. there exist \( \xi \in (c, d) \) s.t. \( f'() = 0 \). A contradiction \( \square \)
Suppose that $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$. If

$$\lim_{x \to a^+} f'(x) = C$$

What can you conclude about $\lim_{x \to a^+} \frac{f(x) - f(a)}{x-a}$?

**Claim:** $\lim_{x \to a^+} \frac{f(x) - f(a)}{x-a} = C$

**Proof:** By MVT, $\frac{f(x) - f(a)}{x-a} = f'(\xi(x))$ for some $\xi(x) \in (a, x)$.

Then $\lim_{x \to a^+} \frac{f(x) - f(a)}{x-a} = \lim_{x \to a^+} f'(\xi(x)) = \#$

Given that $\xi(x) \in (a, x)$, it is clear that $\lim_{x \to a^+} \xi(x) = a$

Thus $\# = \lim_{\xi \to a^+} f'(\xi) = C$. □