Below are a few practice problems for the first midterm. They are intended to be slightly longer and/or harder than the exam problems (the theory being if you can master these problems then the exam should be a piece of cake).

1. Show that
\[ f(x) = \begin{cases} 
\sin(1/x) & x \neq 0 \\
0 & x = 0 
\end{cases} \]
is not continuous at \( x = 0 \) but
\[ g(x) = \begin{cases} 
x \sin(1/x) & x \neq 0 \\
0 & x = 0 
\end{cases} \]
is continuous at 0 (Hint: the sine function is bounded by 1 in absolute value).

2. Prove that there is at least one \( x \in \mathbb{R} \) such that \( 2^x = 2 - 3x \).

3. Given an example of a function \( f : \mathbb{R} \to \mathbb{R} \) such that \( f^2 \) is continuous but \( f \) is not.

4. Show that if \( f : [0, \infty) \to \mathbb{R} \) is continuous and
\[ \lim_{x \to \infty} f(x) = L < \infty, \tag{1} \]
then \( f \) is bounded. Give a counterexample to show that the hypothesis (??) is necessary.

5. Show that if \( f : \mathbb{R} \to \mathbb{R} \) satisfies the inequality \( |f(x)| \leq x^2 \) for all \( x \in \mathbb{R} \), then \( f(0) = 0 \), \( f \) is differentiable at \( x = 0 \), and \( f'(0) = 0 \).