**Legendrian Knots & Lagrangian Fillings**

**R. Casals (UC Davis) @ MSU Geometry & Topology Seminar**

### Introduction

**Def:** The standard contact structure on $\mathbb{R}^3$ is the 2-plane field $\xi_{\mathbb{R}^3} = \ker \{ dz - y dx \}$. It compatifies to $(\mathbb{S}, \xi_\mathbb{S})$.

**Def:** A knot $K \subset (\mathbb{S}, \xi_\mathbb{S})$ is LEGENDRIAN if $TK \subset \xi_\mathbb{S}$.

### Lagrangian Fillings

**Def:** A Lagrangian filling $L \subset (\mathbb{S}, \xi_\mathbb{S})$ is an embedded (nearly) Lagrangian surface in $\mathbb{S}^3$ with boundary $\partial L = K$. Can be a Legendrian, $\partial L = TK$.

**Select Facts:**
1. $\partial L$ might not have a Lagrangian boundary.
2. Legendrians minimally intersect $K$.
3. The $\mathbb{S}^3$-Lagrangian up to Hamiltonian isotopy.

### Classification

**Thm:** (Casals 2020) The Lagrangian twist links $N_{n, m} \subset (\mathbb{S}, \xi_\mathbb{S})$ have

- Infinitely many distinct Lagrangian fillings $\{N_{n, m}\}$
- All $N_{n, m}$ are Legendrian isotopic to $N_{1, m}$.

In fact, for $m = 3$, many Legendrian and satellites knots with this property.

### 2020 Development

**Thm:** (Casals 2020) There exists an abundance of Stein fillable $\mathbb{C}^n$-knots isomorphic to $\mathbb{S}^3$ with

- Only one Lagrangian isotopy class of genus $g$.
- (up to Lagrangian isotopy of genus $g$).

**Subsequently:**
- Legendrian-Seidel conjectures (Casals, Casals),
- Contact transformations & modularity (Casals, Casals),
- Closing the contact picture (Casals, Casals),
- And more! All these are challenges for further research and ongoing studies.
The End

Thank you!