University of California Davis **Name (Print):** ______________________

Differential Equations MAT 22B **Student ID (Print):** ______________________

Midterm Examination **October 25 2019**

Time Limit: 50 Minutes

This examination document contains 10 pages, including this cover page, and 5 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Consider the following first-order differential equation:

$$y'(t) - 5y(t) = e^{5t}.$$

   (a) (10 points) Find *all* solutions to the differential equation.

(b) (5 points) Find the solution $y_0(t)$ to the differential equation satisfying $y_0(0) = 4$.

(c) (5 points) Compute the long-term behavior of *all* solutions $y(t)$.

2. (20 points) Consider the following first-order differential equation:

$$y'(t) = y(y-1)(y+1)(y-3)(y-5)^2.$$

   (a) (7 points) Find the constant solutions of the above differential equation and classify them into *stable*, *unstable* and *semistable*.

(b) (7 points) Let $y_1(t), y_2(t), y_3(t)$ be the solutions of the above differential equation which satisfy $y_1(0) = 2$, $y_2(6) = 3$ and $y_3(0) = 4$, respectively. Compute their long-term behaviors

$$\lim_{t\to\infty} y_1(t), \qquad \lim_{t\to\infty} y_2(t), \qquad \lim_{t\to\infty} y_3(t).$$

(c) (6 points) Use Euler's method with step $h = 0.1$ to approximate the value of $y_3(0.2)$, where $y_3(t)$ is the solution to the differential equation above satisfying $y_3(0) = 4$.

3. (20 points) Consider the second-order differential equation:

$$y''(t) + 9y'(t) + 20y(t) = t + 2.$$

(a) (10 points) Find *one* solution to the differential equation.

(b) (5 points) Find *all* solutions to the differential equation above.

(c) (5 points) Consider the damped harmonic oscillator given by

$$y''(t) + 9y'(t) + 20y(t) = 0.$$

Determine whether the system is *underdamped*, *critically damped* or *overdamped*.

4. (20 points) Consider the second-order differential equation:

$$2t^2y''(t) + 3ty'(t) - 15y(t) = 0, \quad t > 0.$$

(a) (5 points) Show that $y_1(t) = t^{-3}$ is a solution to the above differential equation.

(b) (10 points) Find a second solution $y_2(t)$ which is linearly independent with $y_1(t)$.

(Continued blank space for solution to Part (b))

(c) (5 points) Find *all* solutions to the following differential equation:

$$2t^2y''(t) + 3ty'(t) - 15y(t) = 15, \quad t > 0.$$

5. (20 points) For each of the ten sentences below, circle whether they are **true** or **false**.

(a) (2 points) An autonomous first-order differential equation must have as many stable constant solutions as unstable constant solutions.

(1) True. (2) False.

(b) (2 points) If $y_1(t)$ and $y_2(t)$ solve a first-order differential equation, then their sum $y_1(t) + y_2(t)$ also solves the same differential equation.

(1) True. (2) False.

(c) (2 points) Solutions to underdamped homogeneous systems must have infinitely many zeroes.

(1) True. (2) False.

(d) (2 points) If $y_1(t) = \sin(3t)$ solves a linear second-order differential equation, then $y_2(t) = \cos(3t)$ also solves the same differential equation.

(1) True. (2) False.

(e) (2 points) The local error in Euler's method with step 0.01 is of the order of 0.0001.

(1) True. (2) False.

(f) (2 points) An autonomous first-order differential equation cannot have infinitely many semistable constant solutions.

(1) True. (2) False.

(g) (2 points) The initial value problem given by a second-order differential equation on $y(t)$ and two initial conditions $y(t_0) = y_0$ and $y(t_1) = y_1$ always has a solution.

(1) True. (2) False.

(h) (2 points) The Wronskian of $y_1(t) = e^{-t}$ and $y_2(t) = e^t$ is constant equal to 2.

(1) True. (2) False.

(i) (2 points) The global error in Euler's method is smaller than the local error.

(1) True. (2) False.

(j) (2 points) The Wronskian of $y_1(t) = 1 - t$ and $y_2(t) = t^3$ is $W(y_1, y_2) = t^2$.

(1) True. (2) False.