University of California Davis **Name (Print):** ____________________
Differential Equations MAT 22B **Student ID (Print):** ____________________

Sample Midterm Examination **October 25 2019**
Time Limit: 50 Minutes

This examination document contains 6 pages, including this cover page, and 5 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Consider the following first-order differential equation:

$$y'(t) + 2ty(t) = t.$$

(a) (10 points) Find *all* solutions to the differential equation.

(b) (5 points) Find *all* solutions to the differential equation which satisfy $y(0) = 1$.

(c) (5 points) Compute the long-term behavior of *all* solutions $y(t)$.

2. (20 points) Consider the following first-order differential equation:

$$y'(t) = y(1 - 2y) - 6y + 12, \quad t \geq 0.$$

(a) (10 points) Find the constant solutions of the above differential equation and classify them into *stable*, *unstable* and *semistable*.

(b) (5 points) Suppose that $y_1(t)$ is the unique solution of the above differential equation such that $y_1(5) = 0$. Compute $\lim_{t\to\infty} y_1(t)$.

(c) (5 points) How many solutions $y(t)$ are there with $y(0) = 0$ and $y(10) = 2$?

3. (20 points) Consider the following differential equation:

$$y'(t) = e^{t^2}.$$

In Part (c) you are allowed to leave the expression with terms such as e^2, $e^{-0.1}$, without having to evaluate them.

(a) (5 points) Show that no solution to the differential equation above converges to 0 in the limit $t \to \infty$. Is there any constant solution ?

(b) (5 points) Is there a solution $y_1(t)$ with $y_1(0) = 4$ and $y_1(2) = 1$?

(c) (10 points) Let $y_2(t)$ be the unique solution such that $y_2(0) = 0$. Approximate the value of $y_2(0.2)$ by performing Euler's method with step $h = 0.1$.

4. (20 points) Consider the second-order differential equation:

$$y''(t) + 3y'(t) - 4y(t) = 16te^{3t}.$$

   (a) (10 points) Find *one* solution to the differential equation.

   (b) (10 points) Find *all* solutions to the differential equation above.

5. (20 points) For each of the five sentences below, circle the unique correct answer.

(a) (2 points) The following is a solution of $y''(t) = -4y(t)$:

(1) e^{4t} (2) e^{2t} (3) $\sin(4t)$ (4) $\sin(2t)$

(b) (2 points) All constant solutions to the differential equation $y''(t) = y^3(t) - y(t)$ are:

(1) $0, 1$ (2) 0 (3) There are no constant solutions. (4) $0, 1, -1$

(c) (2 points) The system given by $y''(t) + 4y'(t) + 4y(t) = 0$ is:

(1) Underdamped. (2) Critically Damped. (3) Overdamped.

(d) (2 points) The long-term behaviour of the solutions of $y''(t) + 4y'(t) + 3y(t) = 0$

(1) Does never exist. (2) Exists for some solutions but not for all solutions.

(3) Always exists but depends on the solution. (4) It is always zero.

(e) (2 points) The global error in Euler's method with step h is proportional to:

(1) h^{-1} (2) h (3) h^2 (4) $\ln(h)$